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Design of a hydrostatic symmetric-pad bearing with the membrane-type restrictor

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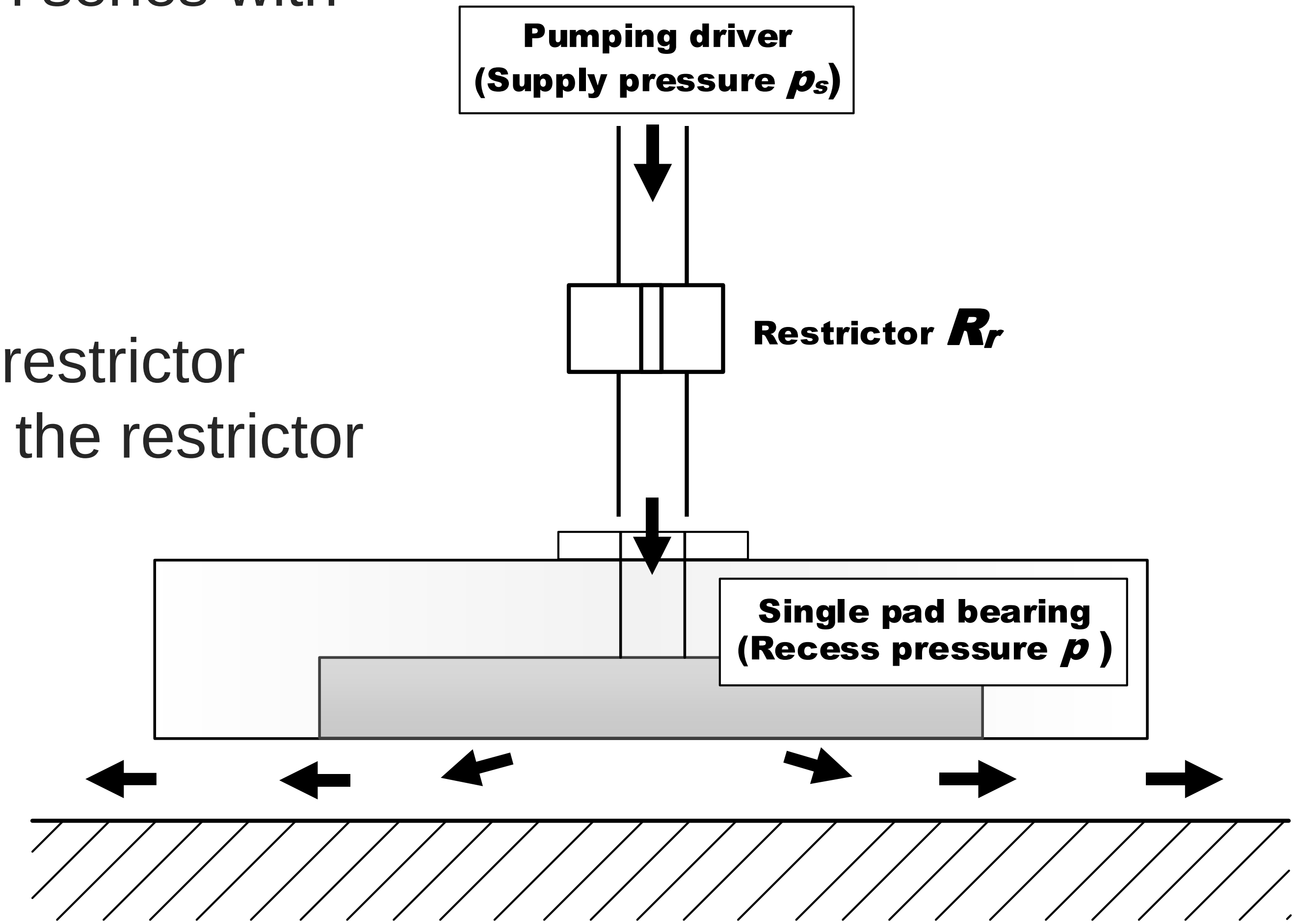
Design of a hydrostatic symmetric-pad bearing with the membrane-type restrictor

Advantages can be characterized for hydrostatic bearings

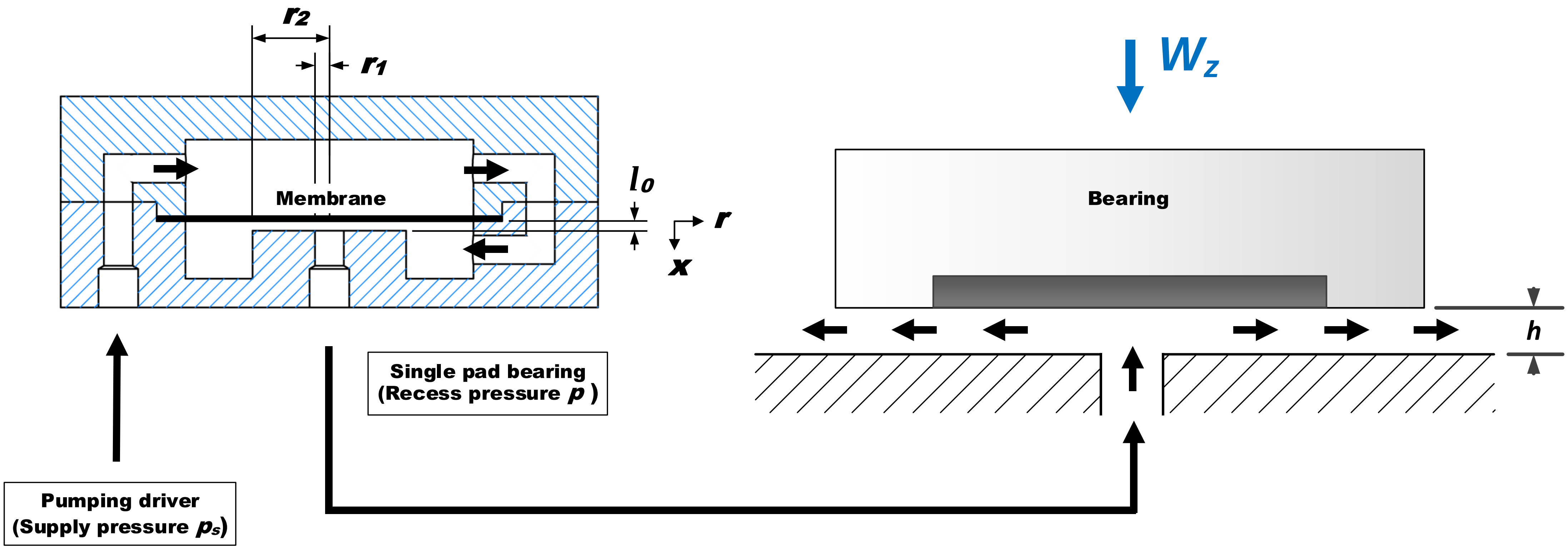
- Low friction (low speeds)
- Infinite life (wear free)
- Zero static friction (no stick slip)
- High damping capacity (squeeze film)
- High stiffness (membrane compensation)

- Most of compensation mechanisms for hydrostatic bearing use a fluid resisting placed in series with each bearing recess.

- When the bearing gap close
  > resistance goes up
  > dropping the flow rate through the restrictor
  > reducing the pressure drop across the restrictor
  > increasing the recess pressure



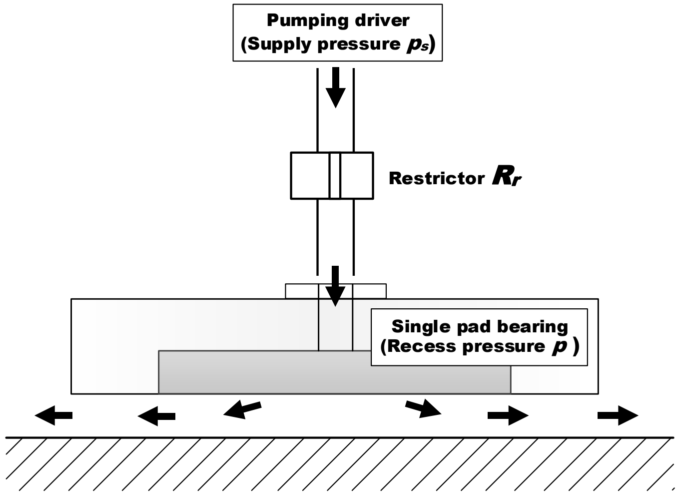
A membrane restrictor differs from many other variable restrictors in that the flow is regulated by a metal diaphragm.
(also called diaphragm controlled restrictor, DCR)



- The flow rate model of the single-pad configuration

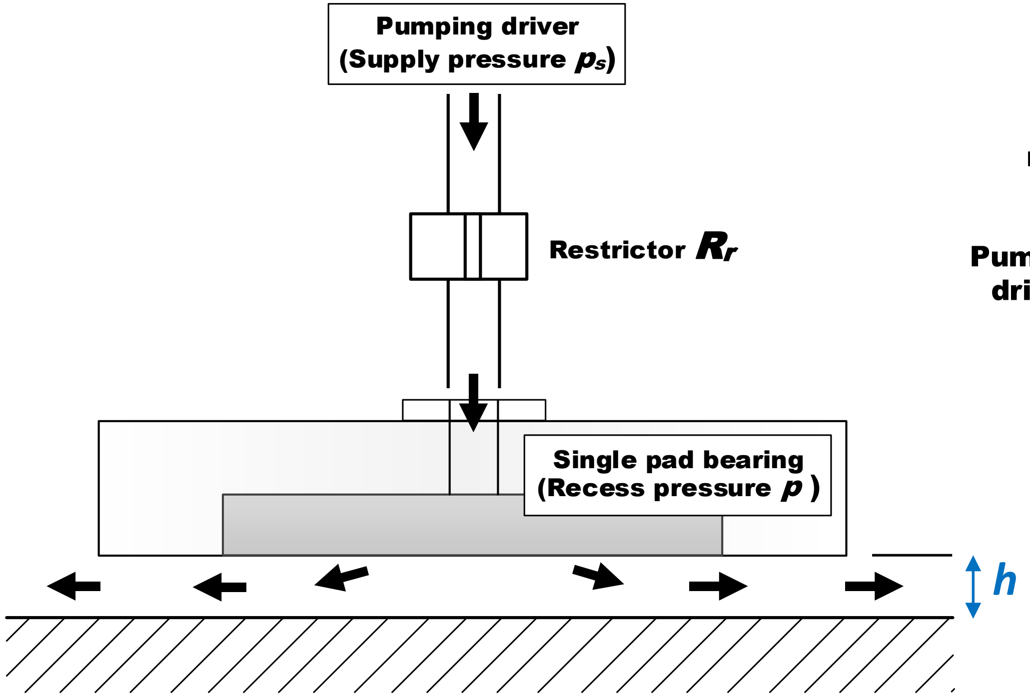
$$Q = \frac{p}{R} = \frac{\Delta p}{R_r\left(\Delta p\right)}, \qquad \Delta p = p_s - p$$

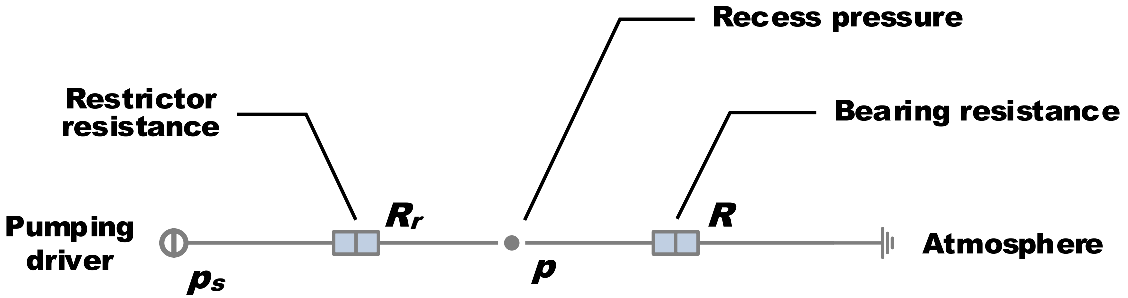
Where *R* is the flow resistance of the bearing, which is inversely proportional to the cube of the clearance *h.*



$$R = \frac{\eta}{h^3} R^*$$

- For the hydrostatic bearing with infinite static stiffness, the flow rate *Q* provided by the restrictor should be linearly proportional to the recess pressure *P*. (i.e. *R(h)* will be a constant)





Analytical consideration for infinite stiffness

When the supply pressure p_s is known, the recesses pressure can be expressed as a function of the resistance ratio R_r/R.

$$p = \frac{p_s}{1 + R_r / R}$$

According to a cubic law of the flow resistance, deformation-pressure relationship of the restrictor can be obtained:

$$p = \frac{p_s}{1 + \frac{R_{ri}}{R}\frac{1}{(1-\xi)^3}}$$

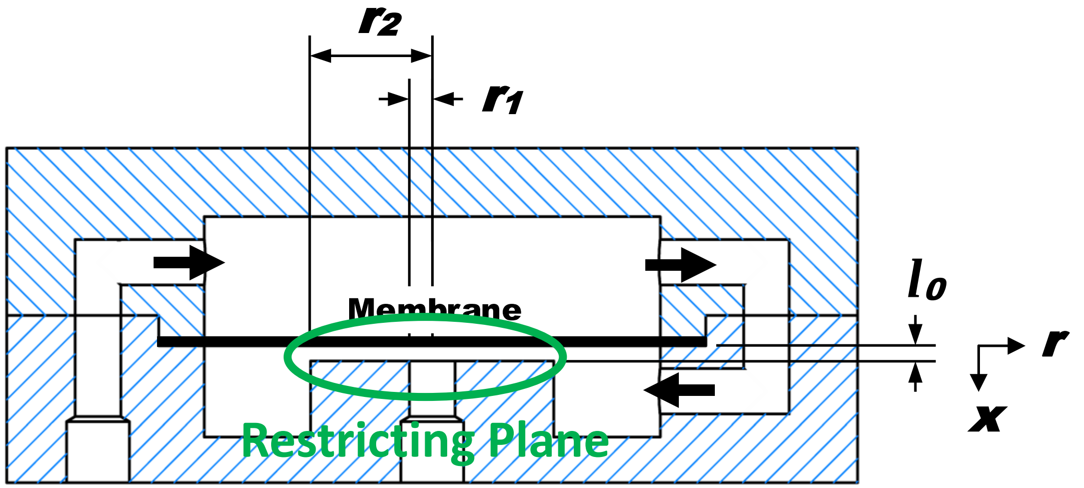
Where ξ is the deformation ratio of the membrane (x/ℓ_0)

For a membrane, the corresponding stiffness can be described as

$$K_r = \frac{d}{dx} A_r \left(p_s - p \right) = -\frac{A_r}{\ell_0} \frac{d}{d\xi} p$$

A_r is the effective area of the restricting plane (considering as a circular pad)

$$A_r = \frac{1}{2} \pi r_2^2 \frac{1-\left(r'\right)^2}{\ln\left(1/r'\right)}, \qquad r' = \frac{r_1}{r_2}$$



The corresponding stiffness of the ideal membrane for hydrostatic bearing with infinite stiffness should be

$$K_{r,i} = 3\frac{p_s A_r}{\ }\lambda\left[(1-\xi)^2 + \lambda\frac{1}{\ }\right]^{-2} = \frac{p_s A_r}{\ }K^*.$$

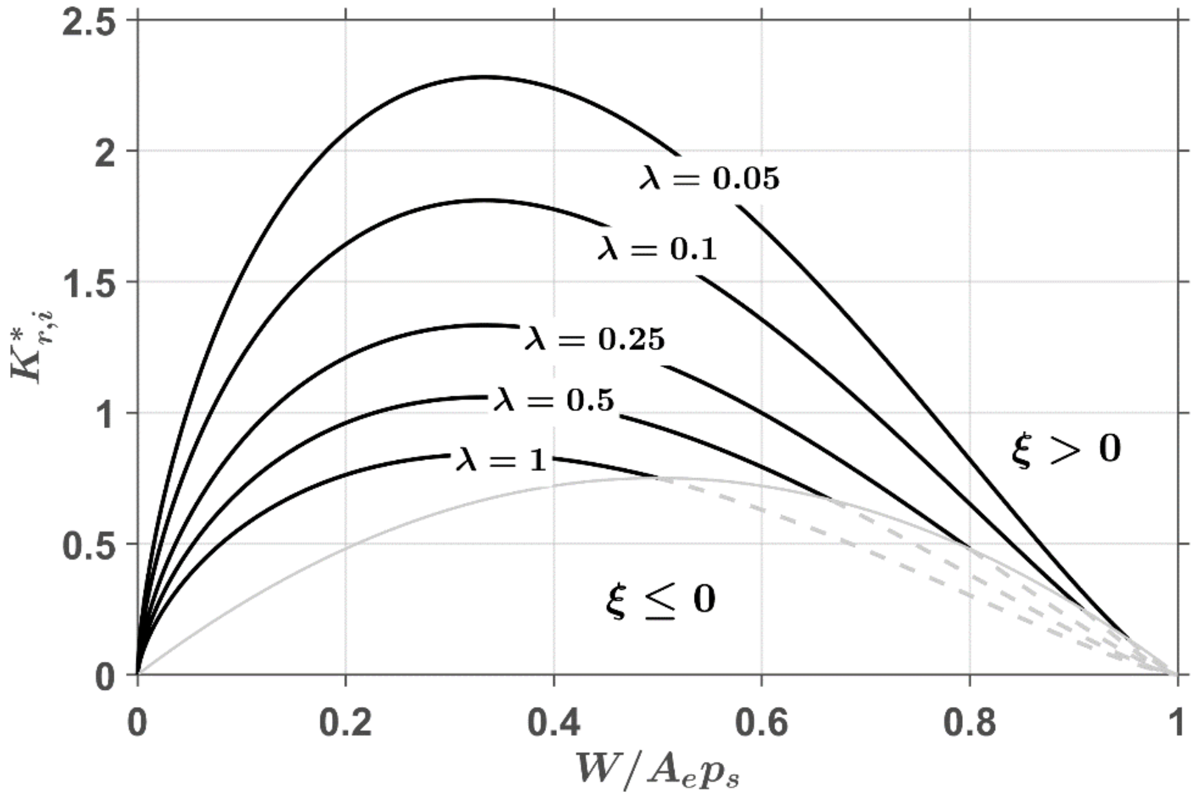
[redacted]

Where λ is denoted as the **design restriction ratio** of the restrictor

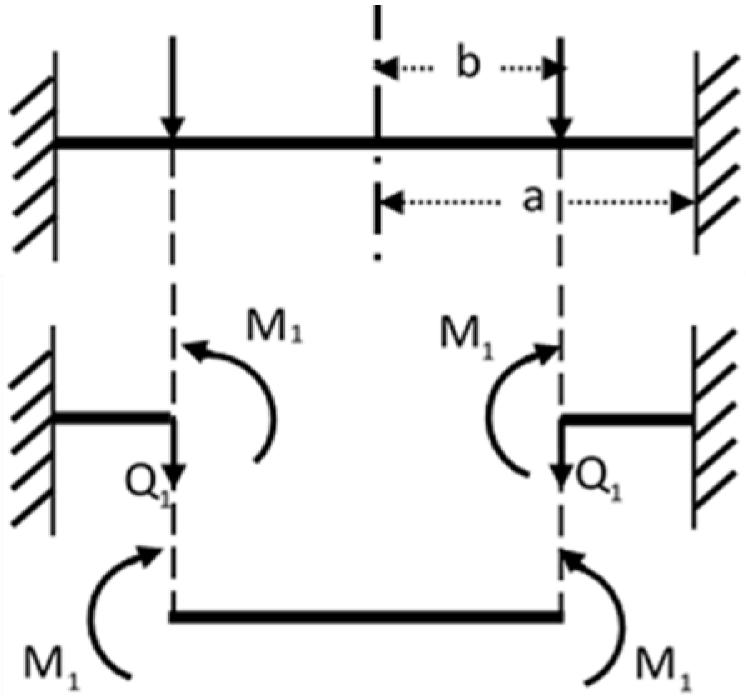
$$\lambda = \frac{R_{ri}}{R_0} = \frac{R_{ri}}{R_{r0}}\left(\frac{1-\beta}{\beta}\right), \qquad R_{ri} = \frac{\eta}{\ell_0^{\,3}}\frac{6}{\pi}\ln\frac{r_2}{r_1}$$

To find a membrane to perfectly satisfy the dimensionless stiffness required for achieving the infinite bearing stiffness is difficult.

$$K_{r,i}^{*} = 3\lambda\left[\left(1-\xi\right)^{2} + \lambda\frac{1}{\left(1-\xi\right)}\right]^{-2}$$



The stiffness of the membrane can generally be assumed as a constant K_r^* by the theory of plates and shells.



Krieger, S. (1959). *Theory of Plates and Shells* (2 ed.). New York: Mcgraw-Hill College.

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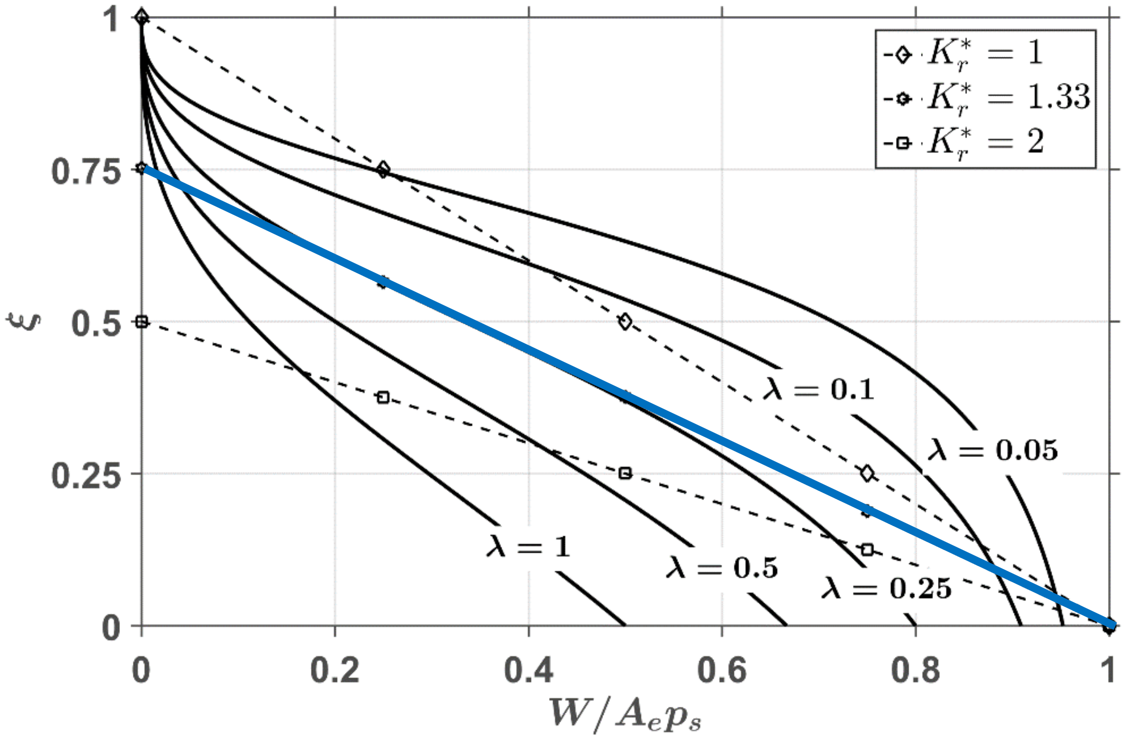
It was found that K_r^* =1.33, λ=0.25 make the deformation-load relationship of the membrane almost compliance with the ideal trend.



Lai TH, Chang TY, Yang YL, Lin SC, Parameters design of a membrane-type restrictor with single-pad hydrostatic bearing to achieve high static stiffness, *Tribology International*, 107(2017), pp. 206-212.

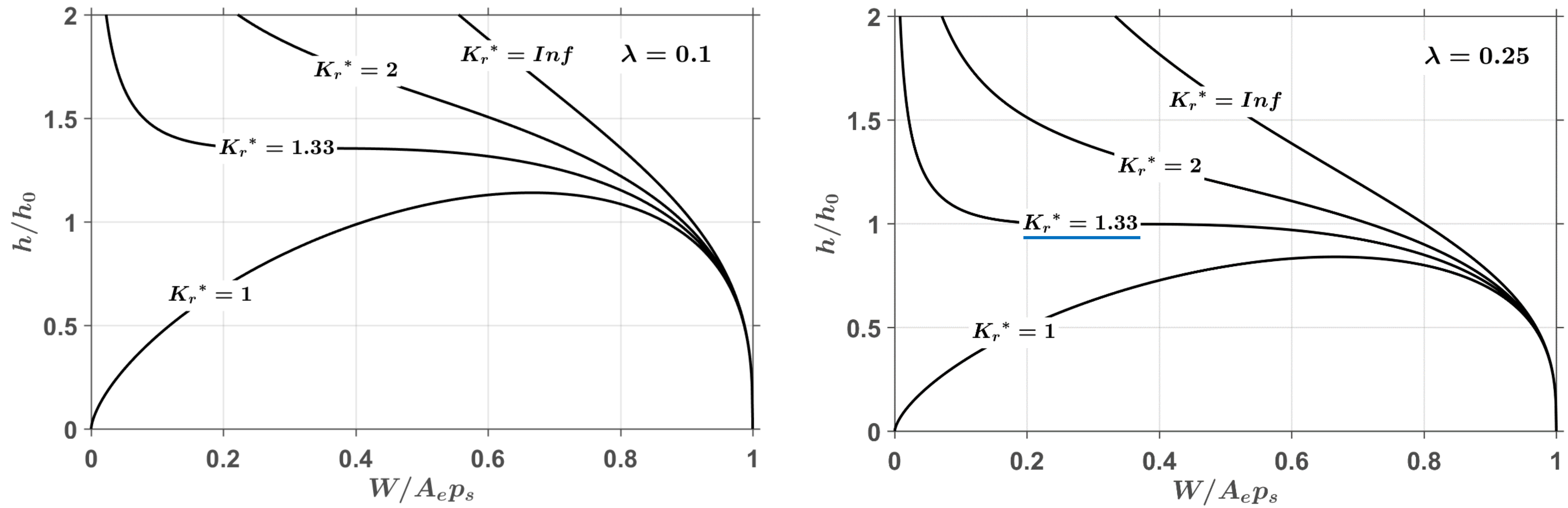
For the single-pad configuration, the clearance of the bearing can be

$$\frac{h}{h_0} = \sqrt[3]{\frac{1-\beta W/W_0}{(1-\beta)W/W_0}}\left[1-\frac{1}{K_r^*}\frac{1}{1-\xi_0}\beta\left(1-W/W_0\right)\right]$$

and the stiffness of the bearing is

$$K = 3\frac{W}{h}\left[\frac{1}{1-\beta W/W_0} - \frac{3\beta W/W_0}{K_r^*\left(1-\xi_0\right)-\beta\left(1-W/W_0\right)}\right]^{-1}$$

Analytical Results and Experiments



When $\lambda = 0.25$, $K_r^* = 1.33$, the bearing will perform very high stiffness at a designed clearance h_0.

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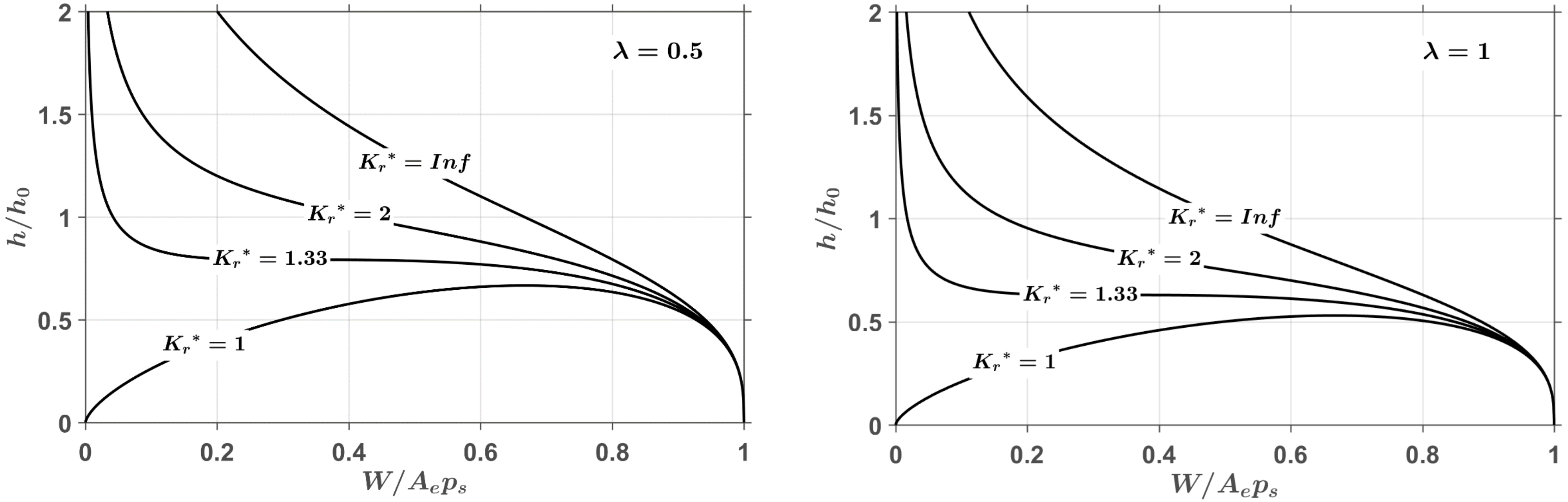
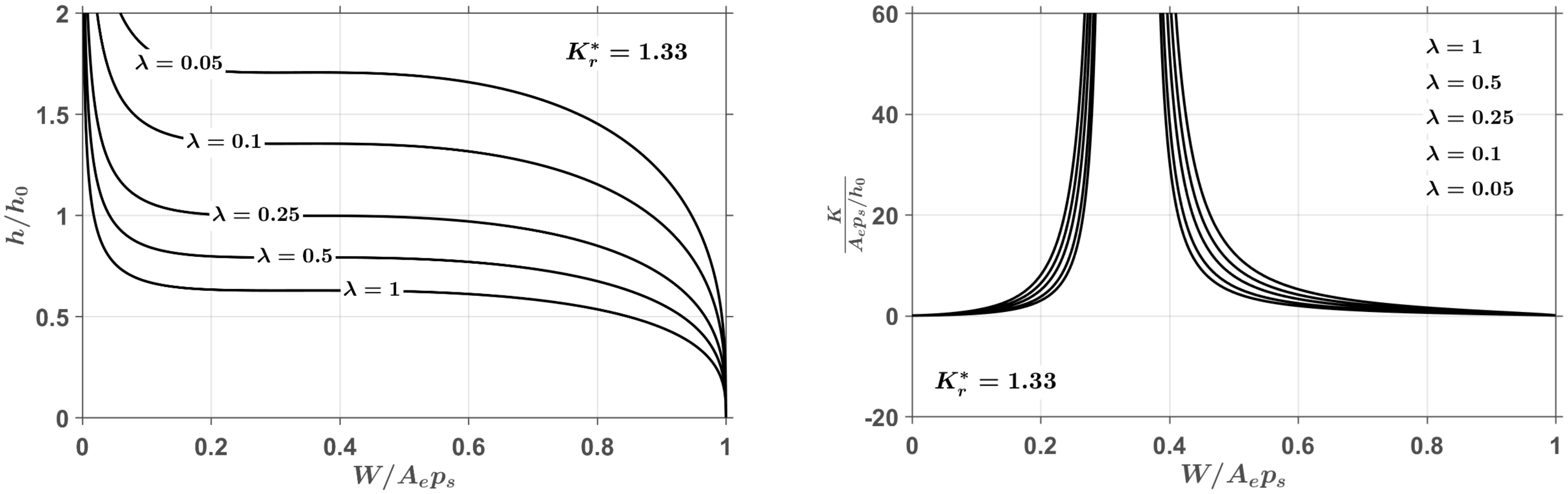
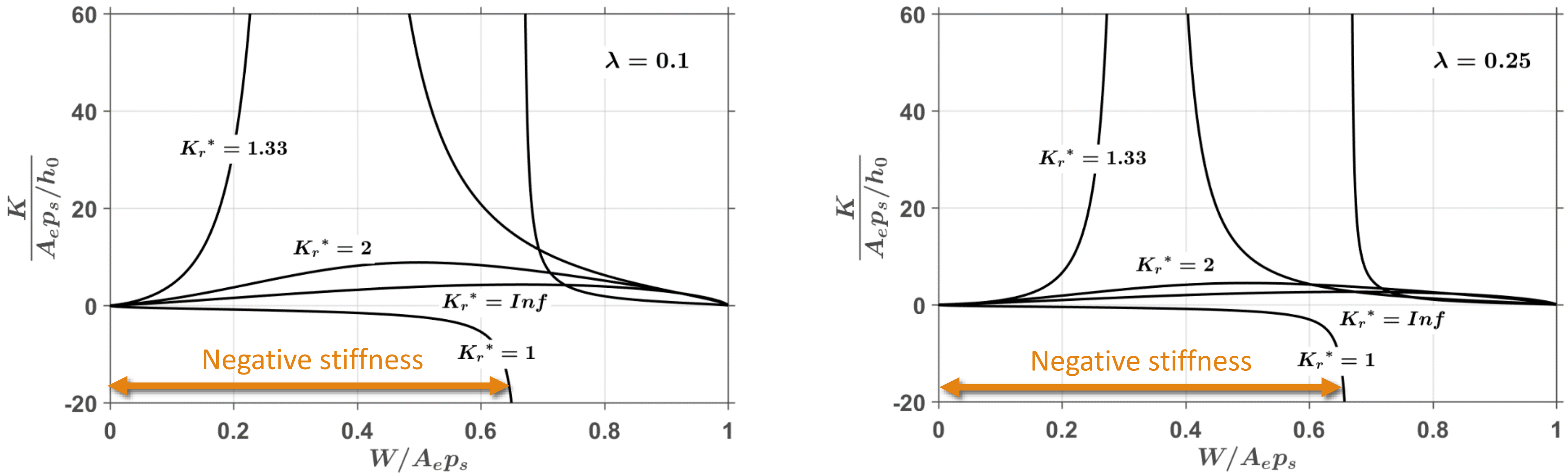


Similar characteristics will observed with different K_r^*, and the clearance ratio will decrease when λ is increased.

Curves for physical membrane with a dimensionless stiffness of 1.33 can achieving the very high bearing stiffness around the region for the dimensionless loading between 0.2 to 0.5.

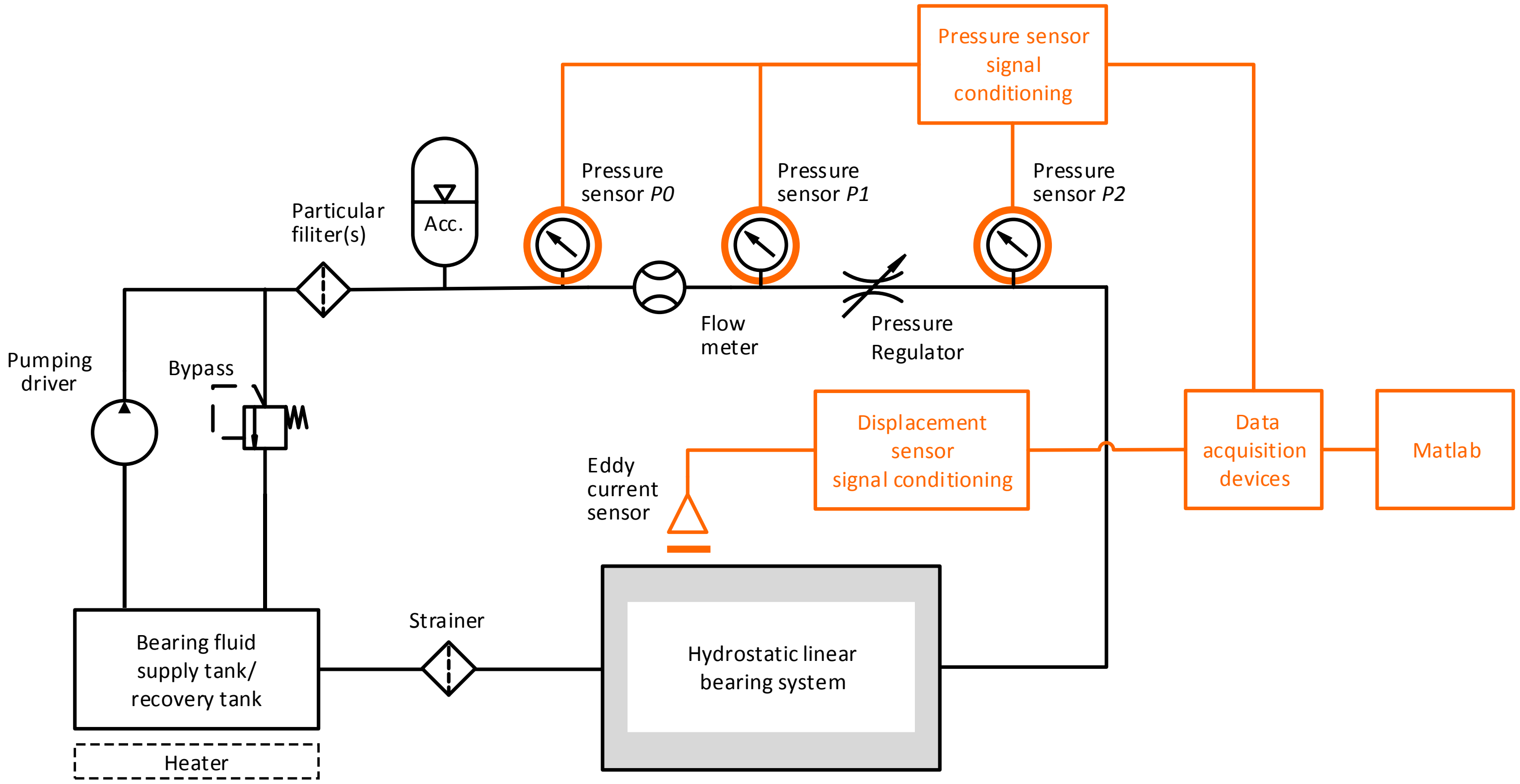


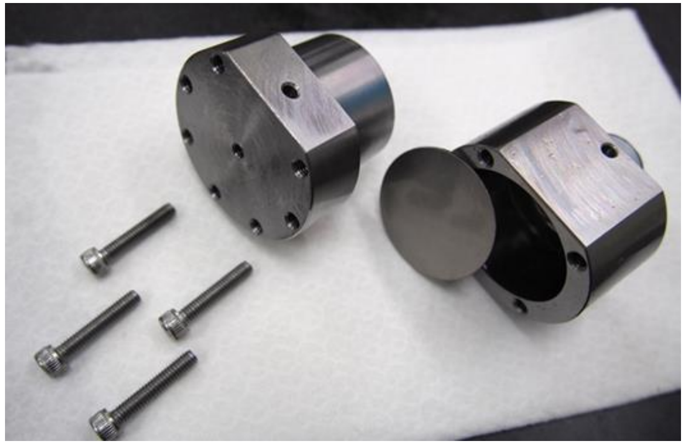
- A very great stiffness could be obtained when $K_r^* = 1.33$, but more consideration is required to avoid the negative bearing stiffness. (when $K_r^* < 1.33$)

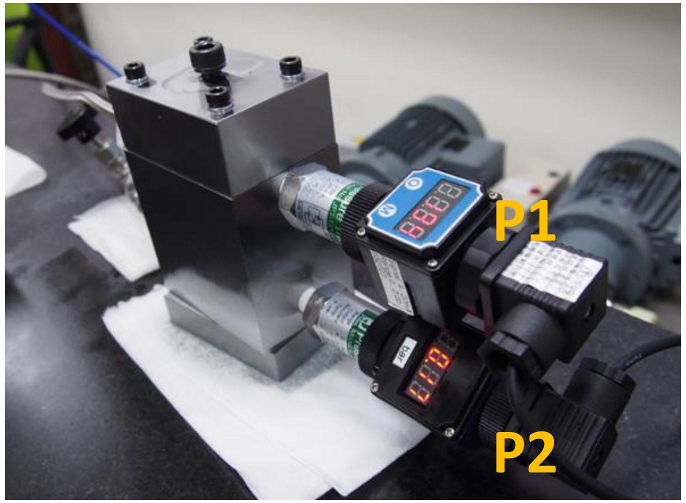


- In the case $K_r^* = 1.0$, the bearing will function with the negative stiffness within the load range $W/A_e p_s = 0{\sim}0.67$

Experiment system diagram





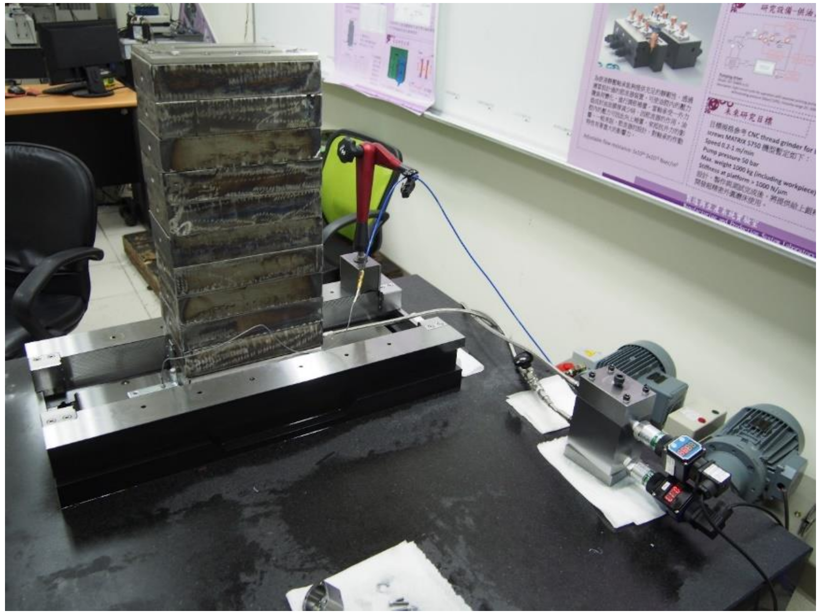


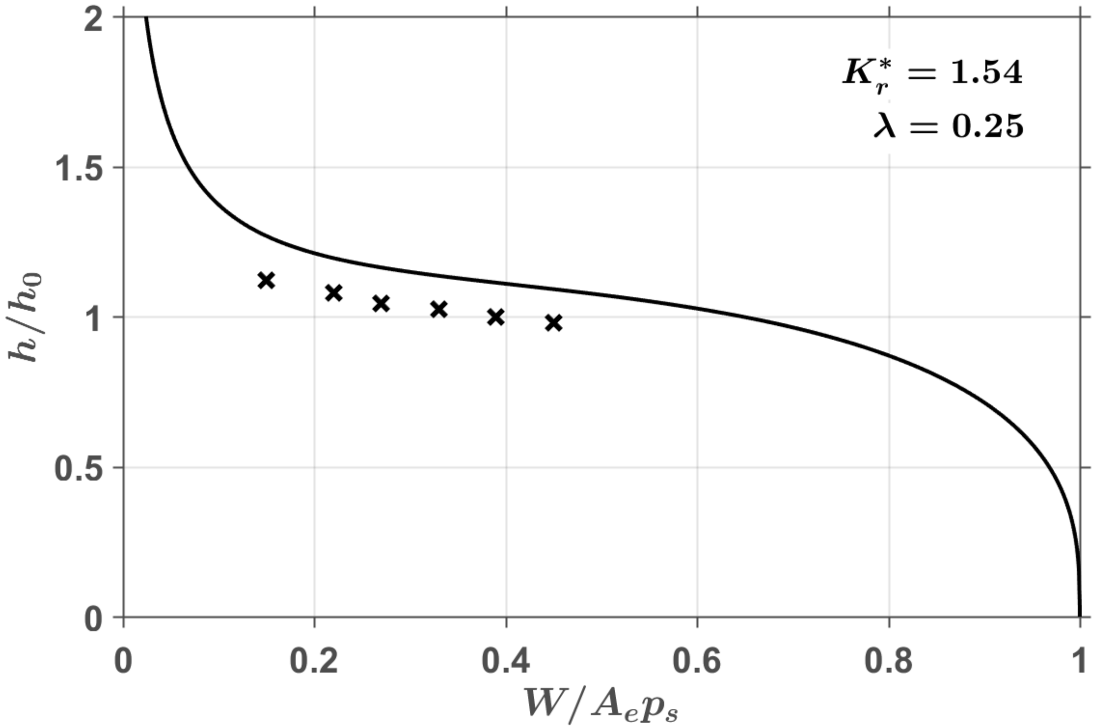
Parameters used in the experiment

Supply pressure p_s	2.0 Bar
Reference pressure ratio β	0.5
Lubricant Viscosity η	0.041 PaSec
Diameters of restrictor r_1, r_2, r_3	6, 10, 15 mm
Designed clearance of restrictor ℓ_0	32 µm
Young's modulus of membrane E	210 GPa
Poisson ratio of membrane v	0.3
Thickness of membrane t	0.5 mm
Applied load W	27, 36, 45, 54, 63, 72 Kg
Size of bearing pad B,C	80,200 mm
Radius of the inner fillet of bearing pad r_i	5 mm
Width of the land of bearing pad ω	30 mm
Reference clearance of bearing pad h_0	20 µm

Chang TY, Yang YL, Liu FR, Lin SC. Analysis on Parameters and Design of Membrane-type Restrictor. Proc of the Int Conf on Engineering and Natural Science - Summer Session; Kyoto; 2016; p.104-115

Max experiment error: -11.8%
Avg experiment error: -10.3%

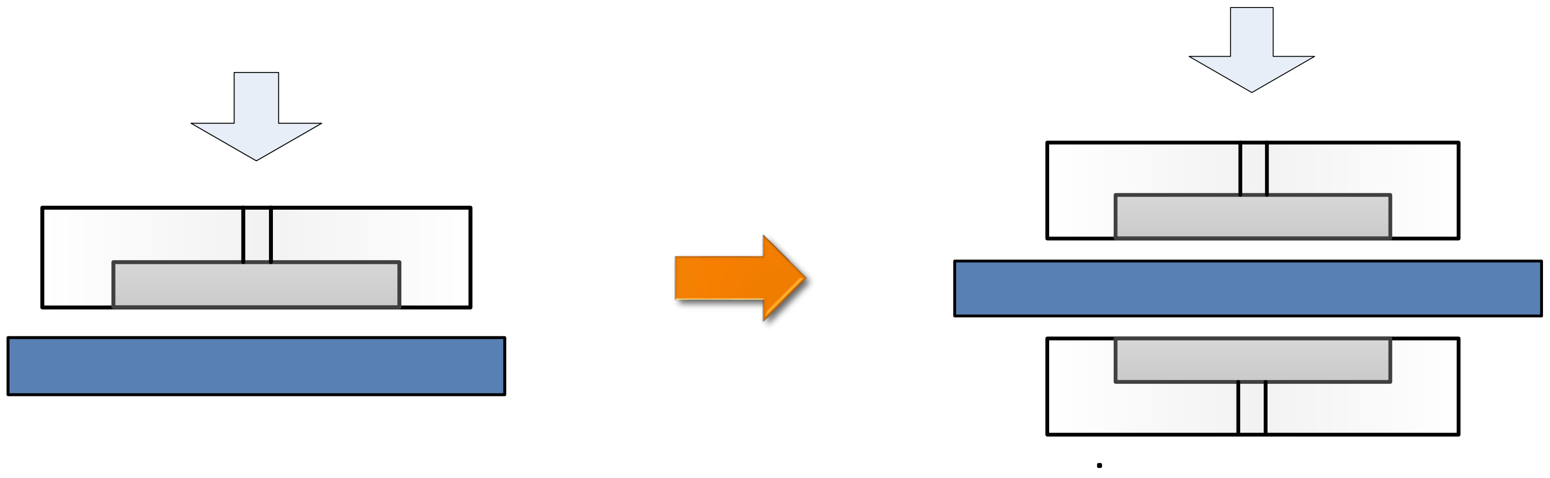




Chang TY, Yang YL, Liu FR, Lin SC. Analysis on Parameters and Design of Membrane-type Restrictor. Proc of the Int Conf on Engineering and Natural Science - Summer Session; Kyoto; 2016; p.104-115

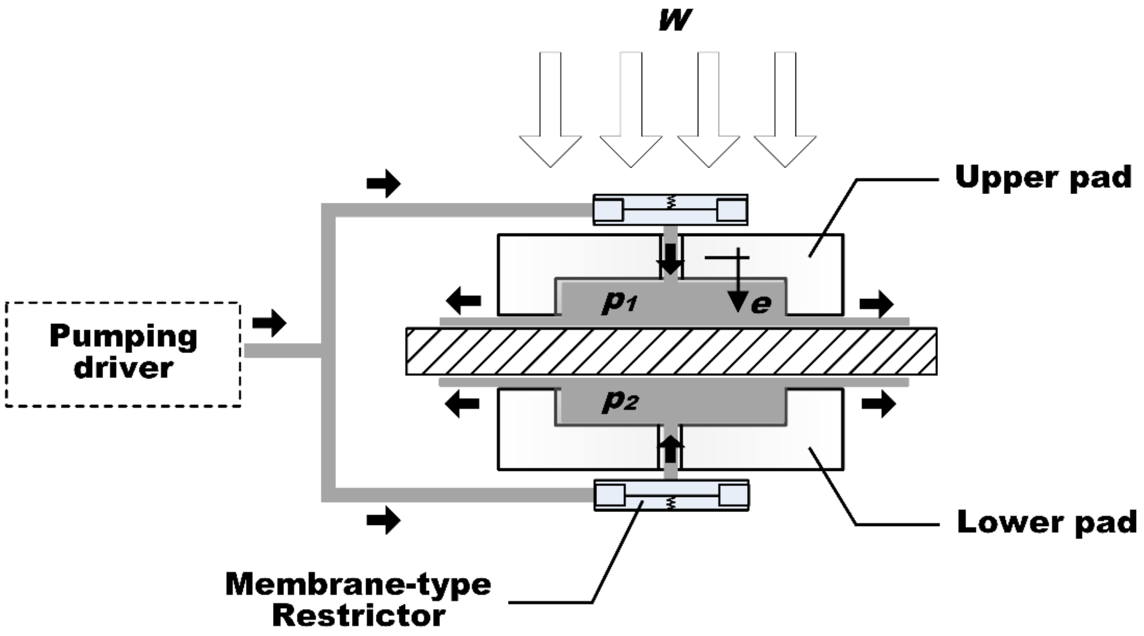
Symmetric-pad bearing with the membrane-type restrictor

In order to get the wider load range and the isolating contaminants capability, the bearing is often designed as a **close form structure** made up of a certain number of **opposed pads**.



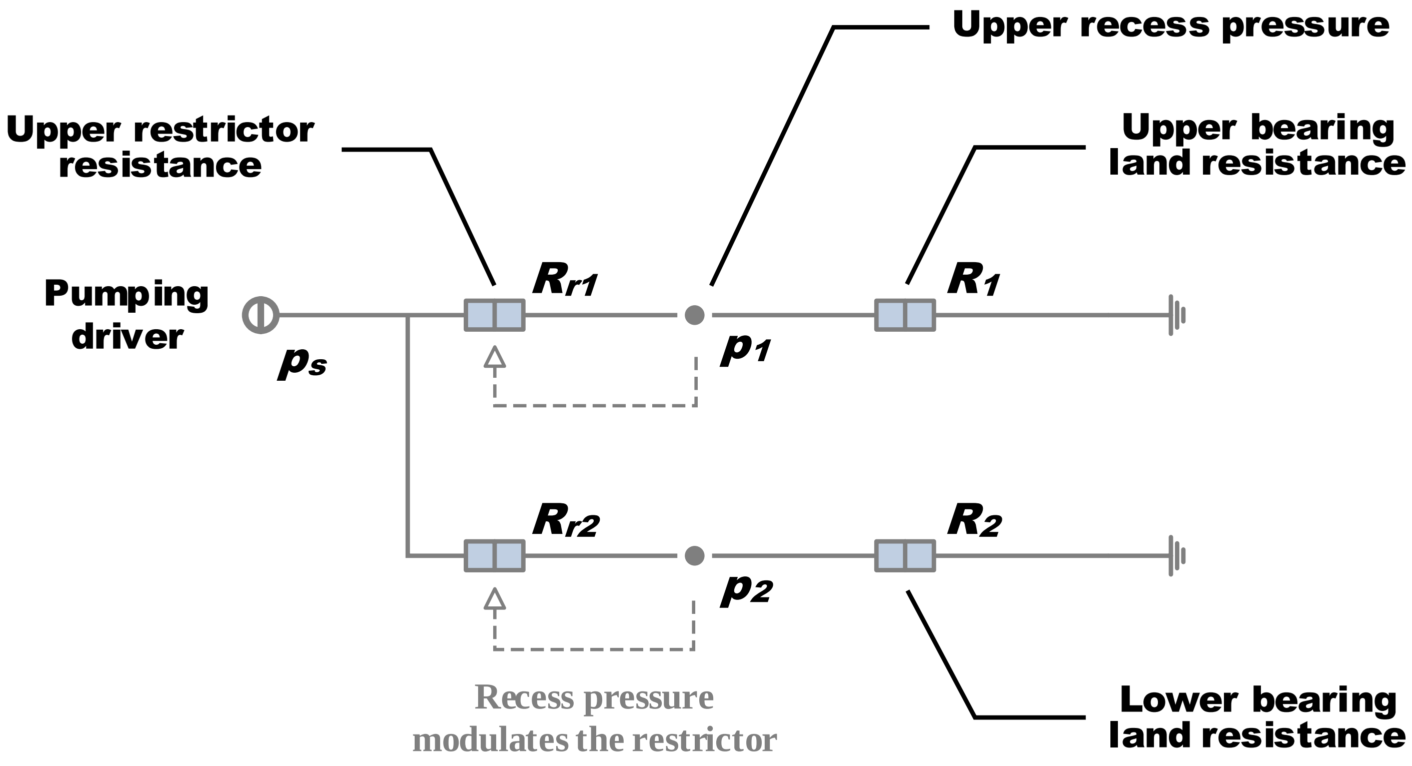
Single pad bearing　　　　Opposed pad bearing

For a opposed-pad bearing, the upper pad and lower pad are equally designed >> A **symmetric-pad** configuration



A sketch plot for a hydrostatic symmetric-pad bearing

The electrical network analogy of the bearing

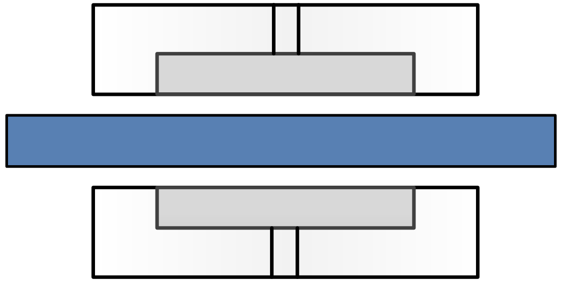


Analytical Results and Discussions

The effects of membrane stiffness on the symmetric-pad bearing were evaluated with the parameter sets listed in the following table

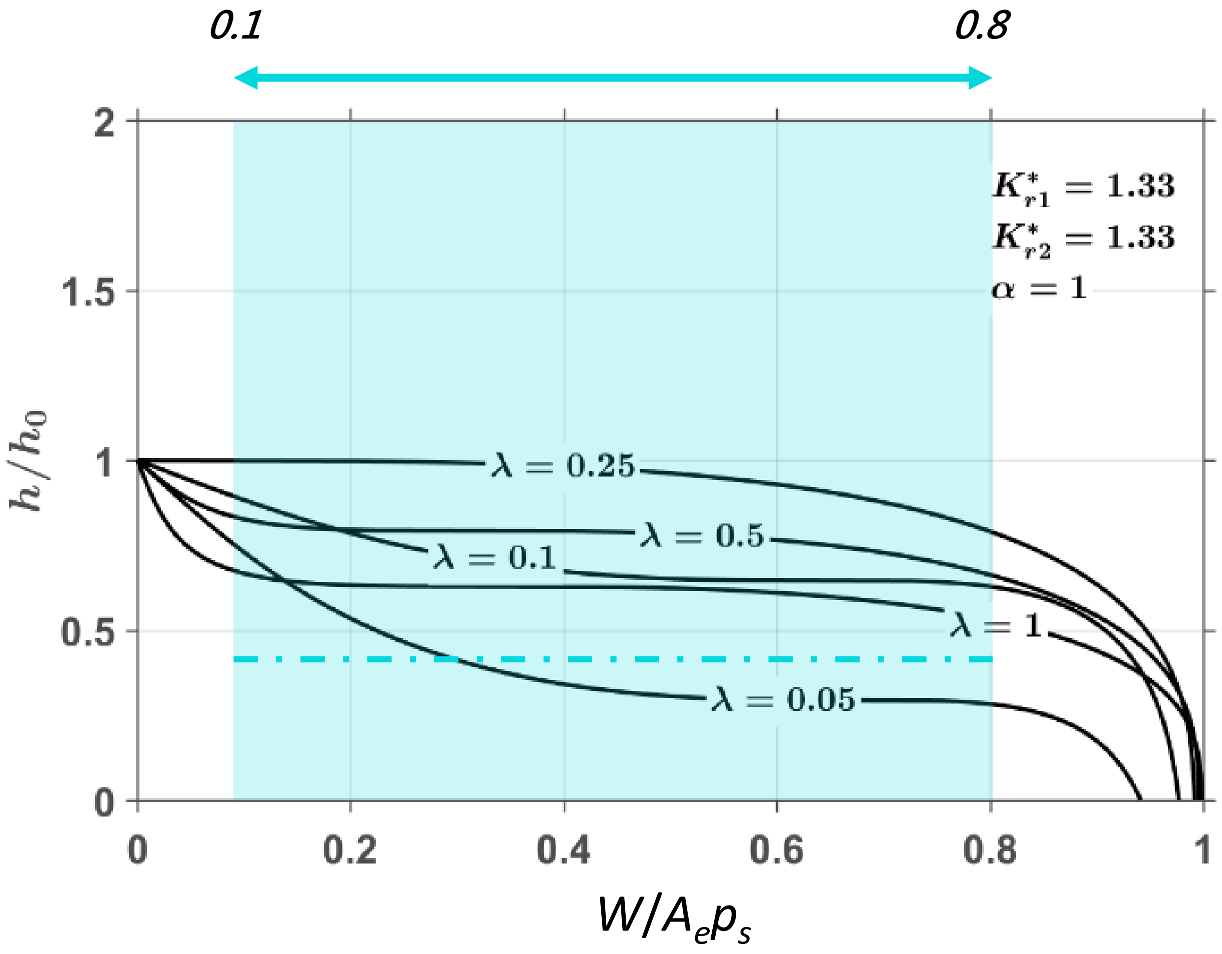
Designation	Values
$K_{r1}^* = K_{r2}^*$	*1.33, 1.50, 2.00, 5.00, Inf*
λ	*0.05, 0.1, 0.25, 0.5, 1.0*
ε	*0 ~ 1.0*

<< Be equally designed



$K_{r1}^* = K_{r2}^*$

Four obvious performance measures were adopt to give the grad of the bearing (ranged from 0.1~0.8 $W/A_e p_s$)



H_{avg} : Average clearance

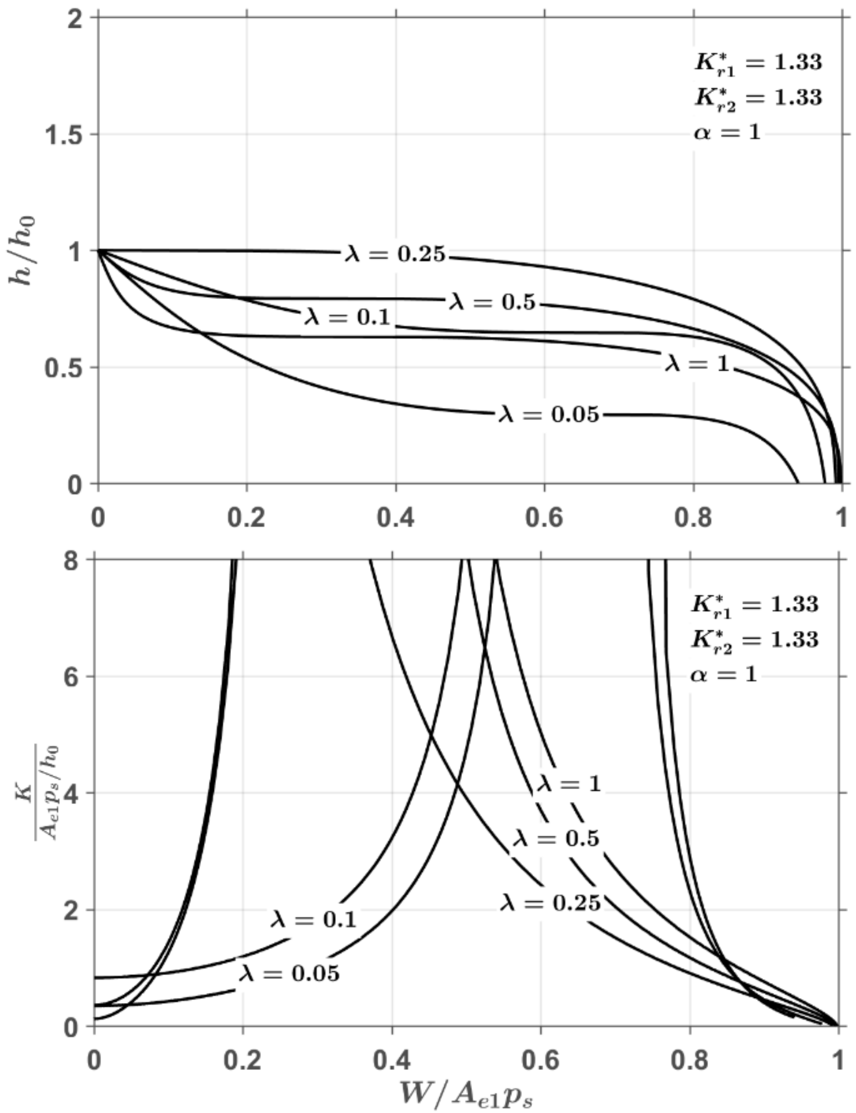
H_{dev} : Deviation of the clearance

H_{dvr} : Deviation ratio of the clearance

H_{min} : Minimum clearance

For the case $K^*_{r1} = K^*_{r2} = 1.33$

λ	H_{avg}	H_{dev}	H_{dvr}	H_{min}
0.05	*.507*	*.444*	*.878*	*.284*
0.1	*.755*	*.252*	*.334*	*.629*
0.25	*.894*	*.206*	*.231*	*.791*
0.5	*.744*	*.162*	*.218*	*.663*
1.0	*.599*	*.132*	*.221*	*.533*
Avg	*.700*	*.239*	*.376*	*.580*



Conclusions

1. The result shows that the membrane-type restrictor has greater static stiffness than traditional ones (i.e. capillary) not only for single-pad cases but also for symmetric-pad configurations.
2. It is recommend to choice K^*_{r1}=K^*_{r2}=1.33 and λ=0.25 for the membrane restrictor in a symmetric-pad bearing for light-load design.
3. The minimal deviation of the bearing clearance will appear when K^*_{r1}=K^*_{r2}=1.33 and λ=1.0 ($\mathrm{i.e.}\ R_{ri} = R_0$). The combination of the design parameters is recommend for heavy-load design.

4. The membrane restrictor will perform as a capillary restrictor when the stiffness of the membrane $K_r^* > 5.0$

5. It is similar to the single-pad case that the static stiffness of the opposed-pad bearing is inversely proportional to the nominal clearance of the bearing. The thinner nominal clearance is, the greater the stiffness and the smaller the power losses will be. (greater manufacturing accuracy will be required)

6. The bearing clearance will be maintained in most application range without any negative stiffness phenomena when both design parameter are properly chosen.

About our lab

Manufacturing and Production System Lab (MAPS)

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Research

Hydrostatic Bearing

- Hydrostatic guideway
- Hydrostatic journal bearing
- Hydrostatic spindle
- Hydrostatic rotary table

Auto-scraping System

- Scraping Mechanism
- Machine Vision
- Image Processing

Professor: Lin, Shih-Chieh

Thanks for listening ~