Load Distribution and Wear-Life Analysis of Aero Involute Spline Coupling

**CATEGORY:** Wear

**AUTHOR AND INSTITUTION:**
- Prof. YUAN Ru, Northwestern Polytechnical University, China
- CHEN Qun, Northwestern Polytechnical University, China
- Prof. ZHAO Ning, Northwestern Polytechnical University, China
- Prof. SUN Linlin, Northwestern Polytechnical University, China

**INTRODUCTION:**
Involute spline couplings are a compact and efficient means for transferring torque between shafts and sleeve in gas turbine aeroengines. Aeroengine spline couplings are complex components that can fail from a variety of mechanisms, and are particularly susceptible to fretting wear and fretting fatigue (FF), which may vitally affect the safety of aeroengines.

In this paper, a statistical distribution of backlash is firstly derived for each tooth in the sequence, along with its average value and standard deviation[1]. Then, an iteration model is derived to describe the radial and axial distribution of the contact pressure.

The geometric changes of spline teeth during fretting wear are taken into account in modified Archard model by an iterative numerical procedure[2]. Finally, the wear-life of involute spline is predicted using the aforementioned numerical model and its load distribution and fluctuations.

1. A statistical distribution of backlash

The teeth on the spline theoretically should have a uniform distribution of backlash. But actually due to machining errors, installation error, etc. The backlash of the teeth in both circumferential and axial is unevenly distributed.

For the purpose of calculating and analyzing conveniently, we can select any one of teeth which named No.1, then number the tooth from 1 to n in a counterclockwise direction. The n-tooth spline is divided into m pieces. So the backlash on both sides of each piece is Eq. (1):
Where $b_{ij}$ is the backlash of internal spline and $b'_{ij}$ is the backlash of external spline, we get Eq. (2):

\[ b_{ij} + b'_{ij} = E - S \]

Where $E$ is the space width of internal spline and $S$ is the tooth thickness of external spline.

Considering that the backlash of spline coupling is random, we can assume Eq. (3):

\[ b_{ij} \geq b_{(n-1)i} \geq \cdots \geq b_{2j} \geq b_{1j} \]

So the backlash of external spline can be described with Eq. (4):

\[ b'_{ij} \geq b'_{(n-1)i} \geq \cdots \geq b'_{2j} \geq b'_{1j} \]

When the spline coupling contact virtually, the backlash of each piece will be Eq. (5):

\[ b_{ij} = b'_{ij} + b_{ij} = E - S \]

We can define a matrix called $b$, consisting of $b_{ij}$ (Eq. (5)) and according to Eq. (6). This matrix describes the backlash of spline coupling.

\[
\begin{bmatrix}
    b_{11} & b_{12} & \cdots & b_{1m} \\
    b_{21} & b_{22} & \cdots & b_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{n1} & b_{n2} & \cdots & b_{nm}
\end{bmatrix}
\]

In the condition of ignoring installation error, we can get Eq. (7) and Eq. (8) from mechanical design manual[3].

\[
E_{\text{max}} - E_{\text{min}} = T \\
S_{\text{max}} - S_{\text{min}} = T \\
\frac{E_{\text{max}} + E_{\text{min}}}{2} = 0.5x_m + \lambda + \frac{T}{2} \\
\frac{S_{\text{max}} + S_{\text{min}}}{2} = 0.5x_m + e_n - \lambda - \frac{T}{2}
\]

Where $T$ is machining tolerance and $\lambda$ is composite tolerance.

It is assumed that $E$ and $S$ satisfy partial normal distribution and Eq. (9) is established, then we get Eq. (10)

\[
\sigma_g = \sigma_s = \frac{T}{6} \\
\mu_g = 0.5x_m + \lambda + \frac{T}{2} \\
\mu_s = 0.5x_m + e_n - \lambda - \frac{T}{2}
\]

If Eq. (9) and (10) are established and then combined, we get Eq. (11):

\[
b_{ij} = E - S - \left( 2x + T - e_n + \frac{T^2}{18} \right)
\]

So we can describe the backlash of spline coupling with statistical model. The wear-life of involute spline is predicted using the numerical model.

2. Iterative model of involute spline coupling wear
n=0

Cycle-index
\( n = n + 1 \)

The new backlash is the original backlash and wear
\( bt_n = bi_{n-1} + ahb_{nt-1} \)

The load distribution is obtained by solving the nonlinear iterative equations
\[
F_n = k \cdot \Delta_n
\]

Calculate the relative sliding distance matrix
\[
TsC_n = f \cdot (TsC_n(:,:,1) - TsC_n(:,:,2))
\]

Calculate single wear matrix of spline
\[
h_{bt_n} = k \cdot TsC_n \cdot \text{aver}(F(:,:,1), F(:,:,2))
\]

Calculate cumulative wear matrix of Spline after nt cycle
\[
a_{hbt_n} = nt \cdot h_{bt_n}
\]

Yes, Break!

Yes, Break!

\[
\max (\max (a_{hbt_n})) \geq \epsilon
\]

\[
nt \cdot n \geq L
\]

No, Loop!

Calculate cumulative wear matrix of Spline after L-nt(n-1) cycle
Fig.1  Spline couplings cycle wear calculation process which consider the effects of geometric

3. Predict the wear-life of involute spline

By the aforementioned iterative algorithm exit conditions (see Fig.1)
\[
\max \left( \max \left( a_{hbt} \right) \right) \geq \varepsilon \tag{12}
\]
\[
nt \times n \geq L \tag{13}
\]

It can be seen from Eq. (12) that out of the loop when the accumulated wear depth greater than required depth of wear \( \varepsilon \). We can obtain the spline couple cycle number when the accumulated wear depth reaches required wear depth. Then compare it with cycle life requirements.

In Eq. (13), we can get the spline couple wear depth when the cycle number reaches required wear life cycle \( L \). Then compare it with required depth of wear.

REFERENCES


KEYWORDS

involute spline, backlash, load distribution, iteration, fretting wear