Elastohydrodynamic Lubrication
With Herschel-Bulkley Model Greases

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The analysis of grease-lubricated rolling element bearings is presented. Experimentally determined flow curves for grease are found to be well correlated by the Herschel-Bulkley model flow equation. A theory for predicting roller film thickness based on the assumed flow model is derived.

Experimental results show that grease will develop a larger film thickness than the grease base oil at first, but the film thickness falls during rolling until it reaches a steady thickness usually lower than that of its base oil. This effect is shown to depend on the degree of shear degradation of the grease, its resulting flow curve, and the temperature rise due to shear in the inlet. The grease yield stress is found to have a negligible effect on EHL performance.

INTRODUCTION
Grease lubrication of rolling element bearings is still more of an art than a science. Most of the standard tests of grease for bearing applications are believed to have low sensitivity or are of secondary importance. Hutton (1) states that the tests are not reliable substitutes for trials under service conditions. A similar viewpoint is presented by Harris (2).

Theories of lubrication under extreme pressure cond-

NOMENCLATURE

- $a$ = stress decay constant
- $b$ = Hertzian half contact width, m
- $D$ = shear rate, sec$^{-1}$
- $E'$ = reduced elastic modulus, N/m$^2$
- $EHL$ = elastohydrodynamic lubrication
- $h$ = roller gap, m
- $h = w_b/(mUR)$
- $h_0$ = gap at zero pressure gradient, m
- $h_p$ = plug flow width, m
- $I(n) = EHL$ film thickness integral
- $K$ = thermal conductivity, Watt/m°C
- $L$ = roller width, m
- $m = 1/n$
- $n$ = power law exponent
- $p$ = pressure, N/m$^2$
- $p' = pressure$ gradient, N/m$^4$
- $p_b$ = maximum Hertz pressure, N/m$^2$
- $q$ = stress decay exponent
- $q_s$ = flow per unit roller width, m$^2$/sec
- $R = reduced roller radius, m$
- $R_{1,t} = roller radius, m$
- $s = surface of roller$
- $t = time, sec$
- $T = temperature, °C$
- $u = velocity, m/sec$
- $n_p = plug flow velocity, m/sec$
- $U = roller surface velocity, m/sec$
- $w = load per unit roller width, N/m$
- $x, y, z =$ coordinates, m
- $\alpha = viscosity pressure coefficient, m^2/N$
- $\alpha_p = Grubin parameter$
- $\alpha q = (\alpha/\rho/2)^{1/2}$
- $\gamma = viscosity temperature coefficient, °C^{-1}$
- $\eta = Newtonian Viscosity, N-sec/m^2$
- $\theta = angle, deg$
- $\lambda = variable$
- $\nu = Poisson's ratio$
- $\tau = shear stress, N/m^2$
- $\tau_p = yield stress, N/m^2$
- $\phi = plastic viscosity, units vary$

Presented at the 27th ASLE Annual Meeting in Houston, Texas, May 1–4, 1972
tions encountered in rolling element bearings and gears have only emerged since the studies of Ertel and Grubin were published in 1949, e.g., Cameron (3) p. 203. As a result of these studies, it was found that it was necessary to take into account the deformation of the bearing surfaces and the increase in viscosity of the lubricant due to the high pressures, e.g., Dowson and Higginson (4). When this was done, the theory predicted that it was possible to have complete fluid film separation of the bearing surfaces on the order of a micrometer under operating conditions. This area of study is called elastohydrodynamic lubrication, abbreviated EHL, and is now well established by experimental verification, e.g., Dyson, Naylor, and Wilson (5).

A literature search was undertaken, but no theoretical EHL studies of grease-lubricated rollers were found. However, Sasaki, Mori, and Okino (6) consider a Bingham plastic flow model of grease lubrication of undeformed rollers. Their results are questionable since they assume the plug flow velocity constant at all parts. A more tractable assumption takes the plug flow velocity constant over any cross section but allows it to vary in the flow direction.

In this paper the selection of an appropriate flow model for grease is considered, and this flow model is then applied to the analysis for film thickness developed under conditions of elastohydrodynamic lubrication of rollers. Theoretical and experimental results are compared to support the assumptions.

RHEOLOGY

Sisko (7) formulated a flow equation for grease of the time-independent power-law type that agreed with experimental results better than the Bingham or Ree-Eyring equations. Bauer, Finkelstein, and Wiberley (8) in their investigations into flow properties of lithium stearate grease modified Sisko’s equation to a form similar to the Herschel-Bulkley (9) equation, and obtained very good correlation with their experiments. Thus, the Herschel-Bulkley flow equation has been selected as the flow model in this paper, and is given by

$$\tau = \pm (\tau_y + \phi |D|^n)$$

Equation [1] implies that grease is a plastic solid; that it acts as a solid until a critical yield stress is reached. Sisko’s experiments support the concept that some fluid flow takes place in the region where “plug flow” would be encountered. The inaccuracy associated with using Eq. [1] is not very severe, as shown by Mahncke and Tabor (10), and the assumption of plug flow is a convenient mathematical approximation well within the accuracy required for most engineering applications. In EHL problems the range of shear stress encountered is so large that the assumption of a yield stress in the flow equation will be shown to be of little consequence to the solution of these problems. Shear rates on the order of 10^4 to 10^5 sec⁻¹ are involved in usual EHL film thickness calculations.

A replot of coaxial cylinder viscometer results for calcium (cup) grease, reported by Vinogradov and Mamakov (11), is given in Fig. 1. The intercept at a shear rate of unity for the plot of \(\tau - \tau_p\) gives the value of the plastic viscosity in Eq. [1], with the slope of the line equal to the power law exponent. The measured data appear to fit Eq. [1] quite well. In Fig. 1, curves A and B show that the shear degradation effect on calcium grease is a significant effect. An interesting feature of this grease is that it shows a shear-thickening effect above about \(D = 1.5 \times 10^4 s^{-1}\), and a shear-thickening effect above this shear rate. Scholten (12) found that a calcium grease was considerably shear thickened at \(D = 7.2 \times 10^4 s^{-1}\) with such erratic results that he did not report the data and suggested that the grease seems unsuitable for bearings operating with thin films. Such a conclusion does not seem to be warranted in light of the results shown in Fig. 1.

Experimental viscometer results for grease presented by Scholten and by Pavlov and Vinogradov (13) approaching shear rates of 10^5 to 10^6 sec⁻¹ support the validity of extrapolating a curve based on Eq. [1] to shear rate levels encountered in EHL film thickness calculations for rollers. However, the extrapolated curve would not be expected to cross over the flow curve for the grease base oil.

EHL THEORY

The two-dimensional flow inlet region of a pair of rollers is shown in Fig. 2. The usual assumptions of negligible side leakage, inertia, gravity, and \(y\) and \(s\) velocity components are taken. The rollers of radius \(R_1\) and \(R_2\) are elastically deformed by an applied load per unit roller width. In real rollers it is usual to find the half Hertzian length much larger than the film thickness, but this plot was designed to fit a hypothetical case discussed later.
As the grease is dragged into the inlet by the moving surfaces of the rollers, a complicated forward-and-reverse flow pattern at low pressure is depicted at the far left. In the region near the load line, where the pressure rise becomes significant, a velocity profile of only forward flow begins, and this is the controlling flow field that will establish the thickness of the lubricant film shown as $h_0$. In the Hertzian pressure zone, parallel flow is assumed although experiments show a small convergence, e.g., (4) p. 153. Along the centerline of the flow field, where the shear stress falls below the yield stress of the grease, a constant velocity at any cross section is shown, known as the plug flow zone. The plug flow should not be confused with plastic flow of a solid, and the plug velocity must increase, as eventually the plug must fill the gap near the centerline of the rollers where the pressure gradient is zero; a boundary condition of the analysis.

It is assumed that the flow is steady, isothermal, incompressible, and laminar. The pressure is assumed to vary only along the flow axis. The Herschel-Bulkley flow equation is assumed to apply to grease with the yield stress and plastic viscosity varying with pressure only; thus, varying in the flow direction only.

The force balance on an element of fluid requires that

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$  \[2\]

Making the usual assumption that the pressure does not vary across the film, Eq. [2] can be integrated to get

$$\tau = \gamma \frac{\partial p}{\partial x}$$  \[3\]

there being no constant of integration since $\tau = 0$ on the centerline, $y = 0$, for the case of pure rolling. Thus, the result is a simple linear variation of shear stress across the inlet, independent of the lubricant properties. According to the Herschel-Bulkley flow model, a central plug flow region occurs in which $\tau < \tau_y$, and enclosed by regions of shear flow (if $\tau > \tau_y$). The plug flow region is of width $h_p$ where

$$\tau_y = \frac{h_p}{2} \frac{\partial p}{\partial x}$$  \[4\]

In the upper shear flow region

$$\phi \left(\frac{d\tau}{dy}\right)^n = \tau - \tau_y = (y - \frac{1}{2} h_p) \frac{\partial p}{\partial x}$$  \[5\]

and, substituting for $d\tau/dx$ from [4], this gives

$$\frac{d\tau}{dy} = \left(\frac{\tau_y}{\frac{1}{2} h_p}\right)^m \left(\frac{\tau_y}{\frac{1}{2} h_p}\right)^m, \quad m = \frac{1}{n}$$  \[6\]

Since the pressure and, therefore, the yield stress and plastic viscosity are assumed to depend on $x$ only, Eq. [6] may be integrated with respect to $y$, and using the boundary condition

$$u = U, \quad y = \pm h/2$$  \[7\]

the equation for the velocity in the upper shear flow region becomes

$$u = U + \left(\frac{2\tau_y}{\phi h_p}\right)^m \left(\frac{1}{m+1}\right) \left\{ (y - \frac{1}{2} h_p)^{m+1} - (\frac{1}{2} h - \frac{1}{2} h_p)^{m+1} \right\}$$  \[8\]

Putting $y = h_p/2$ in Eq. [8], the local plug velocity at the section under consideration is

$$u_p = U - \left(\frac{2\tau_y}{\phi h_p}\right)^m \left(\frac{1}{m+1}\right) \left\{ (\frac{1}{2} h - \frac{1}{2} h_p)^{m+1} - (\frac{1}{2} h - \frac{1}{2} h_p)^{m+1} \right\}$$  \[9\]

Taking advantage of the symmetry of the flow field, the volume flow per unit roller width is calculated from Eq. [8] and [9] to be

$$q_v = 2 \int_0^{h/2} u \, dy = u_p h_p + 2 \left[ U (\frac{1}{2} h - \frac{1}{2} h_p) + \left(\frac{2\tau_y}{\phi h_p}\right)^m \left(\frac{1}{m+1}\right) \left\{ (y - \frac{1}{2} h_p)^{m+1} - (\frac{1}{2} h - \frac{1}{2} h_p)^{m+1} \right\} \right]$$

$$= U h - h_p (U - u_p) - \left(\frac{2\tau_y}{\phi h_p}\right)^m \left(\frac{1}{m+1}\right) \left\{ (\frac{1}{2} h - \frac{1}{2} h_p)^{m+1} - (\frac{1}{2} h - \frac{1}{2} h_p)^{m+1} \right\}$$  \[10\]

In the parallel section near the line of centers of the rollers, the velocity must be uniform and equal to the roller velocity, so that $q_v = U h_0$. For steady flow and if the grease is incompressible, the flow $q_v$ must be constant along the flow path. Substituting for $q_v$ and $u_p$ in Eq. [10] gives

$$U (h - h_0)$$

and

$$= \frac{1}{2(m+1)(m+2)} \left(\frac{2\tau_y}{\phi h_p}\right)^m (h - h_p)^{m+1} (m + 1) h + h_p$$  \[11\]

and raising this to the $n$th power gives

$$\frac{\tau_y h_p}{h_p} \left(1 + \frac{h}{h_p}\right)^{n+1} \left(1 + \frac{n}{n+1} \frac{h}{h_p}\right)^n$$

$$= \phi \left[2 U \left(2 + \frac{1}{n}\right)\right] (h - h_0)^n$$  \[12\]

Consider first the case of a Bingham plastic ($n = 1$). Then, Eq. [12] becomes

$$\frac{\tau_y h_p}{h_p} \left[1 - \frac{3}{2} h + \frac{1}{2} \left(\frac{h_p}{h}\right)^{1/2}\right] = 6 \phi U \frac{h - h_0}{h^2}$$  \[13\]

Substituting for $\tau_y$ from Eq. [4] in the first term only:

$$\frac{d\tau}{dx} - \frac{3\tau_y}{h} \left[1 - \frac{3}{2} \left(\frac{h_p}{h}\right)^{1/2}\right] = 12 \phi U \frac{h - h_0}{h^3}$$  \[14\]
Now, clearly \( \frac{h_p}{h} \) lies between zero and unity, so the term \( [1 - (1/3)(h_p/h)^2] \) lies between unity and two-thirds. Thus,

\[
\frac{dp}{dx} = 12\phi U \frac{h - h_0}{h_0^2} + 2\lambda \tau_v \frac{h}{h_0} \tag{15}
\]

where \( \lambda \) lies between 1 and 3/2, giving a modified Reynolds equation, taking into account the grease yield stress.

For the general case of \( n \neq 1 \), the analysis is more elaborate, but leads to a similar result:

\[
\frac{dp}{dx} = 2\phi \left[ 2U \left( 2 + \frac{1}{n} \right) \right] \left( \frac{h - h_0}{h_0^2} \right)^n + 2\lambda_n \tau_v \frac{h}{h_0} \tag{16}
\]

where \( \lambda_n \) lies between 1 and \( (2n + 1)/(n + 1) \). Omitting the final term gives the result found by Dyson and Wilson (14) for a power law fluid.

Equations (15) and (16) may readily be integrated providing \( \phi \) and \( \tau_v \) depend on pressure in the same way. Unfortunately, nothing about the pressure dependence of either is known. It is possible that the yield stress depends much more strongly on pressure than the plastic viscosity. This would tie in with the Johnson and Cameron discussion (15) of a shear plane mode of failure of oils at very high pressure. In the absence of any data this possibility will not be pursued.

**CASE OF \( \phi = \phi_0 e^\alpha p \) AND \( \tau_v = \tau_w e^\alpha p \)**

For this analysis the usual assumptions of Grubin's theory of EHL are chosen: that the film shape is that given by the Hertzian analysis of the dry contact of rollers, and that the pressure reaches infinitely high values at the beginning of the parallel section.

Crook's (16) approximate form for the film thickness outside the parallel section is taken:

\[
h = h_0 + \frac{21}{3R\phi_0} \left[ |x| - b \right]^{1/2} \tag{17}
\]

where \( b \) is the half width of a Hertzian contact,

\[
b = 2\sqrt{2\sigma R/\pi E'} \tag{18}
\]

where

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}, \quad \frac{2}{E'} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}.
\]

Making the substitution

\[
h = h_0 \sec^2 \theta \tag{19}
\]

it is possible to obtain

\[
|x| - b = \frac{1}{2} (3R\phi_0)^{2/3} b^{-1/3} \sec^{1/3} \theta \tan^{1/3} \theta \tag{20}
\]

To integrate Eq. (15), take out a factor \( e^{-\alpha p} \) to make the left side \( e^{-\alpha p} (dp/dx) \). Then,

\[
\frac{1}{\alpha} (1 - e^{-\alpha p}) = \frac{4}{3} (3R\phi_0)^{2/3} b^{-1/3} \times \int\left[ 6\phi_0 U \frac{\tan^2 \theta}{h_0^2 \sec^2 \theta} + \frac{\lambda \tau_v}{h_0 \sec^2 \theta} \right] \tan^{1/3} \theta \sec^{1/3} \theta \tan \theta d\theta \tag{21}
\]

Assuming that \( e^{-\alpha p} \to 0 \) when \( \theta \to 0 \) gives

\[
\frac{1}{\alpha} = \frac{4}{3} (3R\phi_0)^{2/3} b^{-1/3} \left[ 6\phi_0 U \int_0^{\tau_1/3} \sin^{1/3} \theta \cos^{1/3} \theta d\theta + \frac{\lambda \tau_v}{h_0} \int_0^{\tau_1/3} \sin^{1/3} \theta \cos^{-1/3} \theta d\theta \right] \tag{22}
\]

Substituting \( x = \sin \theta \) reduces the integrals to Beta functions with values

\[
\frac{(2/3)!}{(1/3)!} \text{ and } \frac{(-1/3)!}{(2/3)!} \tag{23}
\]

which, using \( z! = \pi z/\sin \pi z \), and \( z!(-z) = \pi z/\sin \pi z \) gives

\[
\pi/9 \sqrt{3} \text{ and } \pi/\sqrt{3} \tag{24}
\]

[after Jeffreys and Jeffreys (17)].

Therefore,

\[
\frac{1}{\alpha} = \frac{8\pi}{3R\phi_0} \left( \frac{3R\phi_0}{3R\phi_0} \right)^{1/3} \phi_0 U \left[ 1 + \frac{3\lambda \tau_v h_0}{2 \phi_0 U} \right] \tag{25}
\]

Substituting for \( b \) from Eq. (18) leads to

\[
\left( \frac{h_0}{R} \right)^{1/3} = \frac{3.90}{\pi} \alpha \left[ \frac{1}{R} \left( \frac{\phi_0 U}{E' \phi_0} \right)^{1/3} + \frac{3\lambda \tau_v \phi_0}{2 \phi_0 U} \right] \tag{26}
\]

which is Crook's (16) expression for the film thickness with an additional factor of \( [1 + (3\lambda/2)(\tau_v \phi_0/\phi_0 U)] \). Recalling that \( \lambda \) lies between 1 and 1.5, the greatest change in the extra factor is given when \( \lambda = 1.5 \).

For the general case of \( n \neq 1 \), the analysis leads in the same manner to the result given by Dyson and Wilson (14) but with an additional term:

\[
\frac{1}{\alpha} = \frac{4}{3} (3R\phi_0)^{2/3} b^{-1/3} \left[ 2U \left( 2 + \frac{1}{n} \right) \right] I(n) \tag{27}
\]

where,

\[
I(n) = \int_0^{\pi/3} \sin^{2n+1/3} \theta \cos^{2n-1/3} \theta d\theta = \frac{(n - \frac{1}{2})(n - \frac{3}{2})!}{2(n)!} \tag{28}
\]

for \( n > -\frac{3}{2} \).

A plot of \( I(n) \) over the range of interest for grease is given in Fig. 3.
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Solving for \( h_0 \) from Eq. [27] gives

\[
\begin{align*}
  h_0^{n+1/3} &= \frac{2^{3/4} \pi^{1/4}}{3 \sqrt{2}} \left( \frac{E'}{\eta_0} \right)^{1/4} R^{1/2} \sigma \phi_0 U^n \\
  \times \left[ \left( 4 + \frac{2}{n} \right) I(n) + \frac{\lambda \sigma \eta_0 h_0}{\phi_0 U^n} I(0) \right] [29]
\end{align*}
\]

Again, the coefficients are of comparable size, so that the effect of the yield stress depends on the value of \( \left( \sigma \eta_0 h_0 / \phi_0 U^n \right) \). Inspection of the values in Tables 1 and 2 shows that this term is never greater than 0.01. Thus, in all practical cases, the yield stress has a negligible effect on the film thickness.

### LOW YIELD STRESS PRESSURE DEPENDENCE

If the yield stress does not increase with pressure as quickly as the plastic viscosity, then it can only influence the effect of the yield stress depends on the value of \( \sigma_0 \).

#### TABLE 1—ROLLER THEORY AND EXPERIMENT COMPARED FOR A 10% LITHIUM HYDROXYSTEARATE GREASE

<table>
<thead>
<tr>
<th>Grease</th>
<th>Temp. (^{\circ})C</th>
<th>( \eta_0 ) N-sec/m²</th>
<th>Film, ( \mu m )</th>
<th>Rheology ( \tau, N/m² )</th>
<th>Theory/ Test Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. GL3 (19)</td>
<td>35</td>
<td>0.130</td>
<td>2.3, 2.2</td>
<td>(60) 900+42D0.78, 12</td>
<td>1.6</td>
</tr>
<tr>
<td>Li-Soap+90%</td>
<td>35</td>
<td>0.130</td>
<td>2.3, 2.2</td>
<td>(60) 48+1.6D0.81, 0.09</td>
<td>1.1</td>
</tr>
<tr>
<td>Mineral Oil</td>
<td>35</td>
<td>0.025</td>
<td>0.72</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) \( \eta_0 = 2.86 \times 10^8 \) N/m, \( U = 3.68 \) m/sec, \( R = 2.2 \times 10^{-2} \) m, \( E' = 2.27 \times 10^{11} \) N/m², \( \sigma_0 \), \( \sigma_\text{oil} \) = \( (3.57 + 0.985 \log_{10} \phi_0) \times 10^4 \) m²/N.

(b) Little preshear.

(c) Presheared one week in a Klein mill.

(d) Neglects shear heating.

### TABLE 2—THEORETICAL PERFORMANCE OF GREASES IN ROLLERS

<table>
<thead>
<tr>
<th>Grease</th>
<th>Temp. (^{\circ})C</th>
<th>( \eta_0 ) N-sec/m²</th>
<th>Film, ( \mu m )</th>
<th>Rheology ( \tau, N/m² )</th>
<th>Theory/ Test Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1 (12)</td>
<td>25</td>
<td>0.095</td>
<td>1.80</td>
<td>600+1.5D0.84, 1.58</td>
<td>1.56</td>
</tr>
<tr>
<td>Na-Complex Soap</td>
<td>50</td>
<td>0.029</td>
<td>0.68</td>
<td>420+0.55D0.84, 1.56</td>
<td></td>
</tr>
<tr>
<td>+80% Min. Oil</td>
<td>75</td>
<td>0.014</td>
<td>0.35</td>
<td>350+1.2D0.70, 0.74(a)</td>
<td></td>
</tr>
<tr>
<td>No. 2 (12)</td>
<td>25</td>
<td>0.095</td>
<td>1.80</td>
<td>1000+1.15D0.88, 1.91</td>
<td></td>
</tr>
<tr>
<td>Ba-Complex Soap</td>
<td>50</td>
<td>0.029</td>
<td>0.68</td>
<td>600+5.5D0.86, 1.00(b)</td>
<td></td>
</tr>
<tr>
<td>+77% Min. Oil</td>
<td>75</td>
<td>0.013</td>
<td>0.35</td>
<td>450+5.0D0.86, 0.75(c)</td>
<td></td>
</tr>
<tr>
<td>No. 5 (12)</td>
<td>25</td>
<td>0.048</td>
<td>1.03</td>
<td>800+0.37D0.40, 4.38</td>
<td>13.6</td>
</tr>
<tr>
<td>Gel. of Bentonite</td>
<td>50</td>
<td>0.019</td>
<td>0.48</td>
<td>400+0.37D0.40, 6.16</td>
<td></td>
</tr>
<tr>
<td>+90% Synth. Oil</td>
<td>75</td>
<td>0.008</td>
<td>0.23</td>
<td>500+0.044D1.10, 0.07(e)</td>
<td></td>
</tr>
<tr>
<td>No. 6 (12)</td>
<td>25</td>
<td>0.045</td>
<td>0.98</td>
<td>200+0.35D0.85, 1.86</td>
<td></td>
</tr>
<tr>
<td>Clay+75%</td>
<td>50</td>
<td>0.017</td>
<td>0.44</td>
<td>300+0.35D0.75, 1.84</td>
<td></td>
</tr>
<tr>
<td>Synthetic Oil</td>
<td>75</td>
<td>0.008</td>
<td>0.23</td>
<td>280+1.12D0.80, 0.57(c)</td>
<td></td>
</tr>
<tr>
<td>No. 7 (12)</td>
<td>25</td>
<td>0.29</td>
<td>5.20</td>
<td>400+1.3D0.95, 1.44</td>
<td></td>
</tr>
<tr>
<td>Silica+95%</td>
<td>30</td>
<td>0.056</td>
<td>1.14</td>
<td>450+1.6D0.94, 1.88</td>
<td></td>
</tr>
<tr>
<td>Mineral Oil</td>
<td>75</td>
<td>0.018</td>
<td>0.46</td>
<td>80+1.17D0.70, 0.97(c)</td>
<td></td>
</tr>
<tr>
<td>No. 8 (11)</td>
<td>20</td>
<td>0.03</td>
<td>0.71</td>
<td>230+4.3D0.77, 3.95(c)</td>
<td></td>
</tr>
<tr>
<td>Calcium Soap</td>
<td>20</td>
<td>0.03</td>
<td>0.71</td>
<td>570+1.5D0.44, 1.89(d)</td>
<td></td>
</tr>
<tr>
<td>+80% Spin. Oil</td>
<td>50</td>
<td>0.012</td>
<td>0.24</td>
<td>155+3.2D0.75, 4.48(d)</td>
<td></td>
</tr>
<tr>
<td>+1% Water</td>
<td>80</td>
<td>0.005</td>
<td>0.16</td>
<td>3.6+0.045D1.18, 14.5(e)</td>
<td></td>
</tr>
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(a) \( \eta_0 = 2.86 \times 10^8 \) N/m, \( U = 3.68 \) m/sec, \( R = 2.2 \times 10^{-2} \) m, \( E' = 2.27 \times 10^{11} \) N/m², \( \sigma_0 \), \( \sigma_\text{oil} \) = \( (3.57 + 0.985 \log_{10} \phi_0) \times 10^4 \) m²/N.

(b) Presheared at \( D = 7.2 \times 10^4 \text{ sec}^{-1} \), except No. 8.

(c) Presheared at \( D = 5 \times 10^4 \text{ sec}^{-1} \).

(d) Small preshear history.

(e) \( \tau \) (grease) falls below \( \tau \) (base oil) at max. shear rate.
in practice, and no further analysis of this possibility need be considered.

FLOW PATTERN IN THE INLET

If \((\phi/\tau_0)\) is indicative of pressure, Eq. [12] or [13] can be solved to find the variation of the width of the plug flow region. Results are plotted in Fig. 2 for a Bingham plastic assuming the nontypical value for \((\phi U/\tau_0 h_0) = 10\). The plug width narrows to very low values immediately before the parallel section and stays small until the inlet has widened to ten times the minimum width. For larger values of \((\phi U/\tau_0 h_0)\) the plug region becomes too narrow to show. It follows that in Eq. [15] \(\lambda\) must be very close to \(3/2\), and that the factor giving the effect of the yield stress on film thickness from Eq. [26] is very close to \([1 + 2.25(\tau_0 h_0/\phi U)]\).

Ultimately the plug widens to fill the inlet. That is, \(h_p/h \to 1\); but the width of the shear zone \((h - h_p)\) continues to increase.

To find the velocity of the plug flow region, substitute from Eq. [11] into Eq. [9] and get

\[
\frac{w_p}{u} = 1 - \frac{(m + 2)(h - h_0)}{(m + 1)h + h_p}
\]

so that once the variation of \(h_p\) has been found, the local plug velocity is determined and the complete velocity profile easily follows. These are shown in Fig. 2.

It is clear that the plug is not a solid, moving as a rigid body. Although it moves through the parallel section with the velocity of the rollers, further out it moves more slowly, and at large distances it moves backwards out of the inlet. This, of course, exactly parallels the field for oils. An interesting difference is that instead of the backward velocity steadily increasing to \(U/2\) at greater distances, the plug slows down again and ultimately comes to rest—a more plausible behavior than that of oil.

SOLUTIONS COMPARED

Equation [25] can be compared with the known numerical solutions for a Newtonian lubricant as follows. The solutions obtained by Dowson and Higginson (4) follow the relation:

\[
\frac{w_{h_{\text{min}}}}{\dot{\gamma}UR} = 2.1(\alpha q)^{0.60}
\]

where \(q\) \(= w^{1/2}/(\dot{\gamma} U)^{1/2} R\) is proportional to the fluid pressure which would occur in the absence of elastohydrodynamic effects, e.g., Greenwood (18).

Setting \(\tau_0 = 0\) and \(n = 1\) in Eq. [29] there results

\[
\frac{h_0}{R} = 2.21 \left(\frac{\alpha \phi U}{R}\right)^{1/4} \left(\frac{E'}{\dot{\gamma}}\right)^{1/8}
\]

For comparison with Eq. [31], Eq. [32] may be written as

\[
\frac{w_{h_0}}{\phi_0 UR} = 2.78(\alpha q)^{0.50}(\alpha p_0)^{-0.25}
\]

Thus,

\[
\frac{h_0[33]}{h_{\text{min}}[31]} = 1.32(\alpha q)^{-0.10}(\alpha p_0)^{0.25}
\]

For the conditions of Poon’s (19) experiments (Table 1) \(\alpha p_0 = 18.7\); \(\alpha q = 273\) and \(h_0[33]/h_{\text{min}}[31] = 1.57\). Much of this discrepancy is due to comparing the parallel section film thickness with the minimum film thickness, which introduces a factor of about 1.3. Thus, Eq. [29] gives reasonable results for this case, and it appears reasonable to use it to compare the film thicknesses for oil and grease.

EXPERIMENTAL RESULTS

Figure 4 presents roller experiments for a lithium hydroxystearate grease tested by Poon (19). These tests as well as those of Dyson and Wilson (20) generally show that grease will develop a thicker film than its base oil for the same roller conditions at first, but after a short time, the performance of the grease becomes poorer than its base oil. Only grease G1 tested by Dyson and Wilson (20) showed a small increase in film thickness with time, and this was attributed to an increase in viscosity of the grease due to selective evaporation of lighter fractions.

In addition to these roller experiments, microscopic examination of greases confirms the observation that the soap structure is degraded by shear in the rollers such that the original fibrous structure is eventually reduced to small spherical particles of soap in the oil medium.

The effect of shear degradation of calcium grease is clearly evident in Fig. 1. However, the preshear rate of \(5 \times 10^4\) sec\(^{-1}\) for curve \(B\) is far below the maximum shear rate of \(10^6\) to \(10^7\) sec\(^{-1}\) encountered in rollers.

Thus, viscometer and roller experiments make it clear that the length of time of application of shear is important to grease performance, as well as the rate of shear at which shearing occurs.

An estimate of the equivalent percentage of shear degradation for different shear rates can be made using a shear stress decay function proposed by Bauer, Finkelstein, and Wiberley (6), of

\[
\frac{\tau}{\tau_{\infty}} = 1 + \frac{a}{\tau_{\infty}} t^{1-q}
\]

The values of \(a\) and \(q\) did not vary greatly for the greases tested (6), with \(q\) ranging from 0.39 to 0.51 at 37.8 C. Extrapolating the flow curve for the 12 percent lubricant grease at 37.8 C in Fig. 11 (8), and assuming \(q = 0.39\) in Eq. [35], gives

\[
\frac{t(D_1)}{t(D_2)} = \left[\frac{\tau_{\infty}(D_2)}{\tau_{\infty}(D_1)}\right]^{1-16} = \left[\frac{2.5 \times 10^3}{1.1 \times 10^4}\right]^{1.25} = 2.2 \times 10^{-2}
\]

Equation [36] predicts that one hour at \(D_1 = 10^6\) sec\(^{-1}\) is the same as 1.3 minutes at \(D_1 = 10^8\) sec\(^{-1}\). This result is in accordance with the roller and Klein-milled curves shown in Fig. 4, where one week in a Klein mill gives the same results as about 7 minutes in rollers.

Viscometer data presented in Fig. 5 was used to estimate the flow curves for Poon’s (19) GL3 grease. This data was obtained on a Weissenberg cone and plate rheometer taking test points beginning at low shear rates and being careful to avoid grease extrusion effects. The dashed portions represent curve extrapolation.

In Fig. 6, concentric cylinder viscometer data by Schol-
Elastohydrodynamic Lubrication With Herschel-Bulkley Model Greases

GREASE GL3 (19)
Li-SOAP + 90% MINERAL OIL
T = 35°C, OIL FILM = 2 μm

AFTER 1 WEEK IN A KLEIN MILL.

Fig. 4—EHL grease performance in rollers

GL-3 (19)
LITHIUM HYDROXystearate
+ 90% MINERAL OIL

T (A) = 900 + 420D^-0.78
T (B) = 48 + 1·50D^-0.82
KLEIN MILLED 1 WEEK.

Fig. 5—Rheological characteristics of a lithium hydroxystearate grease

BA-COMPLEX SOAP + 77% MINERAL OIL (12)
PRESHRED AT 7.2x10^4 sec^-1
T(25°C) = 1000 ± 250 ± 85
T(50°C) = 600 ± 500 ± 30
T(75°C) = 450 ± 200 ± 30

Fig. 6—Rheological characteristics for barium grease

ten (12) is plotted for a barium grease, with a maximum shear rate of \( D = 7.2 \times 10^4 \) sec^-1. A plot of Eq. [1] in Fig. 6 shows reasonable agreement with the test data.

Table 1 presents a summary based on results from Fig. 5 with theoretical results calculated using Eq. [31] and [29], with test results taken at time zero from Fig. 4. The agreement between theory and test for the grease to oil film ratio is good for the Klein-milled grease, but clearly the isothermal theory is incorrect for virgin grease. Shear rate and temperature rise are examined in the next sections as a possible explanation for the experimental results.

**Shear Rate and Pressure Distribution**

An analysis for the shear rate encountered in the entrance region of rollers with the Herschel-Bulkley flow model of Eq. [1], neglecting the yield stress, is carried out following a similar analysis presented by Dyson and Wilson (14) for a Newtonian fluid.

It can be seen from the flow field sketched in Fig. 2 and from Eq. [3] that the maximum shear rate occurs at the wall of the rollers. Thus, setting \( y = h/2 \) and \( \tau_w = 0 \) in Eq. [5] and rearranging gives

\[
D = \left( \frac{h}{2\phi} \right)^{1/n} \tag{37}
\]

Substituting for \( \rho'/\phi \) from Eq. [16], with \( \tau_w = 0 \), results in

\[
D = 2 \left( \frac{1}{n} + 2 \right) \frac{U h - h_0}{h^2} \tag{38}
\]

On maximizing Eq. [38], it is found that

\[
D = D_{\text{max}} \quad \text{at} \quad h = 2h_0 \tag{39}
\]

Or,

\[
D_{\text{max}} = \left( \frac{1}{n} + 2 \right) \frac{U}{2h_0} \tag{40}
\]

A plot of \( D/D_{\text{max}} \) is given in Fig. 7. The resulting curve is independent of the power law exponent \( n \).

Application of Eq. [40] to the lithium hydroxystearate grease in Table 1, line 1, results in \( D_{\text{max}} = 1.7 \times 10^4 \) sec^-1.

The pressure in the inlet of rollers can be estimated from Eq. [21], with \( \tau_w = 0 \). The result is

\[
\alpha p = -\ln \left[ 1 - \frac{1 - \tau_s}{\tau_s} \right] = \ln \frac{1 - \tau_s}{\tau_s} \tag{41}
\]

The pressure distribution in the inlet is plotted in Fig. 7 along with the shear rate at the wall. The power law exponent has little effect on the pressure distribution, and Fig. 7 shows that the pressure rise is concentrated over a very small distance in the rollers inlet. These curves are
not exact, due to the assumptions leading to Eq. [21], but the trends are in good agreement with more exact calculations (4). A computer solution for \(I_{\text{c}}(\nu)\) was found necessary, as the function has not been tabulated in the range of interest. The values for \(h/h_0\) and \(\nu/\nu_0\) are calculated using Eq. [19] and [20] respectively.

**ROLLER INLET TEMPERATURE**

The fluid passing through the inlet of rollers will experience very high viscous shear, as shown in Fig. 7. Crook (21) shows that heat transfer by convection is small compared to that by conduction, so that to a good approximation the heat produced in each element of fluid can be equated to the heat conducted away from the element, and for a Newtonian fluid

\[
-K \frac{d^2T}{dy^2} = \tau \frac{du}{dy} = \frac{1}{\eta} \left( \frac{dp}{dz} \right) y^2 \quad [42]
\]

Crook solves this equation approximately to obtain an upper limit to the temperature rise. For his condition for oil the values found were only \(\frac{1}{2}\) to 1 degree C. For experimental results Crook states, “The temperatures due to rolling friction were not influencing the film thickness.”

When grease is considered, the flow curves shown in Fig. 1 and 5 indicate that there may be considerable heating in the inlet since the shear stress is everywhere greater than for the base oil at the same temperature. For the properties associated with greases, Crook’s analysis leads to predictions of temperature rise which are unduly high. A simpler and better upper limit for temperature rise can be obtained by noting that since it is motion, not force, which is imposed on the grease, the effect of a reduction in viscosity must be to reduce the temperature rise. Thus, the temperature rise for the case of constant viscosity is an overestimate when the viscosity falls with temperature.

Taking the assumption of constant viscosity at any cross section, Eq. [42] is integrated twice, noting that \(dT/dy = 0\) at \(y = 0\), and the result is

\[
-K T = \frac{1}{\eta} \left( \frac{dp}{dz} \right)^2 y^4 + C \quad [43]
\]

The temperature difference between the center line at \(y = 0\) and the surface of the roller at \(y = h/2\) is, therefore,

\[
T_c - T_s = \Delta T = \frac{1}{K \eta} \left( \frac{dp}{dz} \right)^2 \frac{h^4}{192} \quad [44]
\]

Substituting for \(dp/dz\) from the isothermal Reynolds equation,

\[
\frac{dp}{dz} = 12 \eta U \frac{h - h_0}{h^3} \quad [45]
\]

gives the result wanted:

\[
T_c - T_s = \frac{3}{4} \frac{\eta U^2}{K} \left( 1 - \frac{h_0}{h} \right)^2 \quad [46]
\]

This differs from Crook’s result in the omission of a factor \(e^{-\sigma T}\) from the left side, where \(\gamma\) is the temperature coefficient of viscosity.

The analysis for grease follows precisely that given above (the yield stress \(\tau_y\) is neglected) and leads to

\[
T_c - T_s = \frac{2^{n-1} \left( 2 + \frac{1}{n} \right)^n \phi U^{n+1} \left( k - h_0 \right)^{n+1}}{3 + \frac{1}{n}} \frac{k^2}{H_{\phi}} \quad [47]
\]

The dependence on \(n\) in Eq. [47] is not strong enough for the small deviations from unity found with greases to be significant. But the very much higher values of the plastic viscosity of grease compared to the viscosity of oils means that according to Eq. [47] and data in Tables 1 and 2, temperature rises of 20 to 50 C, and higher, are found in the inlet of rollers using grease.

**THEORY APPLIED**

Scholten (12) presents flow curves for five greases measured with a concentric cylinder viscometer. The greases were tested after being sheared at \(D = 7.2 \times 10^4\) sec\(^{-1}\) until no apparent change in shear stress could be determined. His paper suggests that about an hour was the longest preshear time allowed. Equation [36] would indicate that an hour at a shear rate of \(10^4\) sec\(^{-1}\) would correspond to only a few minutes in rollers where the maximum shear rate will be greater than \(10^4\) sec\(^{-1}\). An example of Scholten’s data for a barium-complex soap with 77 percent mineral oil is shown in Fig. 6 on an In-In plot. The apparent yield stress determined from Fig. 6 is larger than Scholten’s reported yield stress, but his discrepancy is not important since the effect of yield stress on EHL calculations is insignificant, as shown previously.

A summary of flow curve equations and theoretical predictions for a number of greases is presented in Table 2. The theoretical EHL grease/oil film thickness ratio for calcium grease gives an opposite trend from that of the lithium grease in Table 1. The predicted values show that the calcium grease should increase its film thickness with time. It is also interesting to note that although \(\phi_0\) decreases to 1/3 its initial value due to preshear, \(n\) increases from 0.63 to 0.77 to produce a thicker film. Unfortunately, no viscometer shear rate trends are available for the other greases listed in Table 2.

The bentonite grease in Table 2 looks significantly better than the other greases as far as grease/oil film thickness ratio, but based on the results shown in Fig. 4 for the lithium grease, it must be concluded that roller tests are necessary before any steady-state performance expectations could be predicted.

Table 2 includes results for several greases where, at the maximum shear rate value in the inlet of rollers, the extrapolated viscometer curve for the grease crosses the curve for the base oil, as indicated by note (5). Dyson and Wilson (20) consider the thickening effect by the suspension of soap in the oil and find that the viscosity of the grease should always be greater than its base oil at the same temperature. With the much greater shear heating of the grease, it is not inconceivable that the viscometer data in these cases is in error. On the other hand, shear
heating in the inlet of rollers has not been included in the theoretical film thickness calculation, so that this data may point to the explanation for the roller results where the grease/oil film thickness ratio falls below unity.

Based on the general correlation between Poon's (19) roller experiments (Fig. 4) and the theoretical predictions (Table 1) using viscometer data (Fig. 5), it is suggested that theory based on viscometer data alone will predict grease performance in rollers. However, further experimental work is needed to establish confidence in applying this suggestion to the many types of greases available.

The results shown in the section on temperature rise due to shear indicate that the film thickness theory can be improved considerably if the isothermal assumption is discarded in favor of a more realistic assumption for the temperature distribution in the inlet of rollers.

CONCLUSIONS

The derived theory presented in this paper may be used for predicting grease elastohydrodynamic film thickness in rollers. Within the limitations of the assumptions, the results appear to be supported by the available experimental evidence.

It is found that most greases behave pseudoplastically, and although their shear stress at a given shear rate does not fall below the shear stress of their corresponding base oil at the same temperature, it may be possible to have a smaller film thickness than the base oil due to the nonisothermal fluid mechanics of the grease in the inlet of the rollers.

Shear thinning of a grease at a given shear rate in a viscometer can be misleading, as in the case of calcium grease, since some greases will shear thicken or shear thin depending upon the shear rate.

The available grease experiments with rollers tend to support the conclusion that the theory plus viscometer data may be used to predict grease performance. However, viscometer data at higher shear rates than currently reported in the literature are needed for EHL predictions.

A nonisothermal solution for film thickness analysis appears to be required in order to explain grease EHL performance with greater accuracy.

ACKNOWLEDGMENTS

The authors wish to express their sincere appreciation to K. L. Johnson, Cambridge University, and A. Dyson, Shell Research Ltd., for their helpful discussions and suggestions on grease EHL. Also, thanks are due to the University of Virginia for its support of the senior author via an appointment as Sesquicentennial Associate of the Center for Advanced Study during 1970–1971.

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