Elastohydrodynamic Grease Lubrication Theory and Numerical Solution in Line Contacts

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In this paper, film thickness and pressure distribution of EHL in line contacts for four kinds of greases and one type of oil are calculated under a series of rolling velocities by using an inverse solution method. The Herschel-Bulkley model is used for the rheological characteristics of the grease. Based on the theoretical analysis and numerical calculation, the following influences on film thickness and pressure distribution in EHL of grease are analyzed: the effects of starvation, yield stress, presheared grease and fresh grease, and approach and separation between two cylinders. The effect of different rheological characteristics of grease on film thickness is also investigated.

INTRODUCTION

The elastohydrodynamic lubrication problem is usually associated with highly stressed machine elements such as rolling bearings, gears, and cams. High quality solutions exist for the EHL theory which describes oil flow with Newtonian characteristics between two cylinders. Since most rolling bearings are lubricated by grease, it is necessary to solve the EHL equations for grease in order to determine the properties of grease film between two cylinders.

The earliest attempt at a numerical solution of EHL for grease was reported by Greenwood and Kauzlarich (1) who used the Herschel-Bulkley model to characterize grease and analyzed EHL for grease by the Grubin theory. A simplified solution was obtained. Sanae Wada (2) also dealt with this problem with an incomplete numerical solution. The solution is problematic because it does not consider the pressure spike in the outlet region. Jonkisz (3) reported a theoretical and experimental investigation of EHL for grease and obtained a group of numerical solutions using an inverse iterative method. However, in his solutions, the Reynolds equation is not complete for the whole contact region and gives little attention to starvation of grease, the squeeze problem, or to yield stress. Therefore, in this paper, these problems are investigated. A computer program is designed combining an inverse iterative method with a forward-iterative method. The Herschel-Bulkley model is used for the rheological characteristics of the grease.

NOMENCLATURE

\[ E' = 2 \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) \text{, equivalent Young's modulus, } \text{Pa} \]
\[ G = \alpha E' \text{, dimensionless materials parameter} \]
\[ H = b/R \text{, dimensionless film thickness} \]
\[ h = \text{film thickness, m} \]
\[ H_0 = h_{0}/R \text{ dimensionless central film thickness} \]
\[ H_{\text{min}} = h_{\text{min}} / R \text{ dimensionless minimum film thickness} \]
\[ \rho = \text{pressure, Pa} \]
\[ R = \text{equivalent radius of cylinders, m} \]
\[ P = \mu E' \text{, dimensionless pressure} \]
\[ Ra = V / U, \text{ dimensionless velocity ratio} \]
\[ U = \left( \frac{n_a}{E'} \right)^2 \frac{U}{R}, \text{ dimensionless velocity parameter} \]
\[ \bar{U} = (U_1 + U_2)/2, \text{ average entrainment rolling velocity, m/s} \]
\[ V = \left( \frac{n_b}{E'} \right) \frac{V}{R}, \text{ dimensionless squeezing velocity parameter} \]

\[ \bar{V} = \text{squeezing velocity, m/s} \]
\[ W = \omega (E' R), \text{ dimensionless load parameter} \]
\[ \eta = \eta / \eta_0, \text{ dimensionless viscosity} \]
\[ \eta_0 = \text{viscosity at ambient pressure, } \text{Pa s} \]
\[ \rho = \text{dimensionless density} \]
\[ \kappa_m = \text{film thickness where } \frac{\partial \eta}{\partial \kappa} = 0 \]
\[ \bar{X} = x a, \text{ dimensionless coordinate} \]
\[ n = \text{rheological parameter of grease model} \]
\[ \gamma_s = \text{grease yield stress} \]
\[ \tau = \text{shear stress} \]
\[ a = \text{half Hertzian length, m} \]
\[ \lambda_n = \text{constant } \lambda_n = 1 - (2n + 1)/(n + 1) \]
\[ \rho R = \text{maximum Hertzian pressure, Pa} \]
\[ \rho_s = \text{pressure of the pressure spike, Pa} \]
\[ \tau_{eo} = \tau_{eo} / E', \text{ dimensionless grease yield stress} \]
BASIC EQUATIONS

The problem considered is that of two elastic cylinders, which roll on each other without sliding and, in some situations, with an approach or separation relative to each other. They are lubricated by grease described by the Herschel-Bulkley model. It is assumed that the flow of grease between the two cylinders is steady, isothermal, incompressible, and laminar. The pressure is assumed to vary only along the flow axis which is the x-direction. The Herschel-Bulkley model equation is assumed to apply to the grease, with the yield stress and grease viscosity (plastic viscosity) varying with pressure only, thus, varying in the flow direction only.

It is assumed that the viscosity-pressure relation of grease is the same as that of base oil, and that viscosity and yield stress depend on pressure in the same way.

The viscosity-pressure relation proposed by Barus is used in this paper:

$$\eta = \eta_0 e^{\alpha p}$$  \[1\]

where $\alpha$ is the piezoviscous coefficient. In this report it is fixed at $2.2 \times 10^{-6} Pa^{-1}$. $\eta_0$ is viscosity at ambient pressure, for grease $\eta_0 = \eta_0$ and for oil $\eta_0 = \eta_{oil}$.

The yield stress/pressure relation can be expressed as:

$$\tau_y = \tau_{y0} e^{\alpha p}$$  \[2\]

where $\tau_{y0}$ is the yield stress at ambient pressure.

From Dowson and Higginson (4), the density of grease can be written as:

$$\rho = \rho_o \left( 1 + \frac{0.58\rho}{1 + 1.7\rho} \right)$$  \[3\]

The Herschel-Bulkley model can be expressed as:

$$\begin{cases} 
\frac{\partial u}{\partial y} = \frac{1}{\eta} (\tau - \tau_y) \quad |\eta| \geq \tau_y \\
\frac{\partial u}{\partial y} = 0 \quad |\eta| < \tau_y 
\end{cases}$$  \[4\]

The force balance on an element of fluid requires that:

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$  \[5\]

For the case of pure rolling, a constant velocity zone, known as the plug flow zone, can be formed. The boundary condition can be written as follows (1):

$$y = y_c/2 \quad \frac{\partial u}{\partial y} = 0$$  \[6\]

where $y_c$ is plug flow width.

From Eqs. [4], [5], and condition [6], it is possible to obtain:

$$\eta \frac{\partial u}{\partial y} = \frac{\partial p}{\partial x} (y - y_c) \quad (y \geq y_c)$$  \[7\]

In the field near the inlet of the contact, where $\frac{\partial p}{\partial x}$ is greater than zero, Eq. [7] can be integrated with respect to $y$ and using the boundary condition:

$$u = \overline{U} \quad y = h/2$$  \[8\]

Thus, the equation for the velocity in the upper shear flow region near the inlet can be written:

$$u = \overline{U} - \frac{n}{n+1} \left( \frac{1}{\eta \frac{\partial p}{\partial x}} \right)^{\frac{1}{n}} \left[ \left( \frac{h - y_c}{2} \right)^{\frac{n+1}{n}} - \left( \frac{y - y_c}{2} \right)^{\frac{n+1}{n}} \right]$$  \[9\]

In the same way, the equation can be obtained for the velocity in the field near the outlet of the contact, where $\frac{\partial p}{\partial x}$ is less than zero.

$$u = \overline{U} + \frac{n}{n+1} \left( \frac{1}{\eta \frac{\partial p}{\partial x}} \right)^{\frac{1}{n}} \left[ \left( \frac{h - y_c}{2} \right)^{\frac{n+1}{n}} - \left( \frac{y - y_c}{2} \right)^{\frac{n+1}{n}} \right]$$  \[10\]

When the condition is $|\eta| < \tau_y$, then $u = \overline{U}$, $0 \leq y \leq y_c$. Integrating Eq. [5] from $-y_c/2$ to $y_c/2$, yields:

$$\frac{\partial p}{\partial x} = \frac{2\tau_y}{y_c}$$  \[11\]

The continuity equation of the fluid is:

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0$$  \[12\]

Substituting Eq. [9] into Eq. [12], integrating Eq. [11] with respect to $x$ and $y$ respectively, and using the boundary condition $x = x_m$, $\frac{\partial p}{\partial x} = 0$, produces:

$$\frac{1}{\eta \frac{\partial p}{\partial x}} \left( \frac{1 - \frac{y_c}{h}}{n} \right) \frac{n+1}{n} \left( 1 + \frac{n}{n + 1} \frac{y_c}{h} \right)$$  \[13\]

where $\overline{V} = -\frac{1}{\rho \partial \tau} (\rho u)$, $\frac{\partial p}{\partial x} > 0$.

In the same way:

$$\frac{1}{\eta \frac{\partial p}{\partial x}} \left( \frac{1 - \frac{y_c}{h}}{n} \right) \frac{n+1}{n} \left( 1 + \frac{n}{n + 1} \frac{y_c}{h} \right) = -2 \left( 2 + \frac{1}{n} \right)$$  \[14\]

where $\frac{\partial p}{\partial x} < 0$. 
In order to facilitate simplification for Eq. [13] and Eq. [14], it is designated: \( b/a = n \), i.e. a real number can be divided into the quotient of two numbers.

Combining Eqs. [11], [13], and [14] and simplifying:

\[
\frac{\partial p}{\partial x} = 2^{(1-n)} \frac{(2 + \frac{1}{n})}{n} \left[ \frac{2\left[ U(h - \eta \rho_0/p) - \bar{V}(x - x_0) \right]}{h^{2n+1}} \right] + 2\lambda \frac{\tau}{\rho \lambda} \frac{\partial p}{\partial x} = 0
\]

where

\[
\lambda_n = 1 - \left( \frac{a}{b} \left( 1 - \frac{\eta}{h} \right) \left[ -\frac{1}{n+1} \left( 1 + \frac{n \eta}{h} \right) \right] \right) + \frac{a \cdot (a-b)}{b \cdot 2b} \left( \frac{\eta}{h} \left[ 1 - \frac{\eta}{h} \right] \left[ -\frac{1}{n+1} \left( 1 + \frac{n \eta}{h} \right) \right] \right)^2 \}
\]

This Reynolds equation is suited to conditions where \( \rho_0 \) is either greater or less than \( \rho_0 \eta \). It is more complete than previous Reynolds equations which describe the lubricated region only by Eq. [13]. When not considering the approach motion, \( \bar{V} = 0 \) in Eq. [15].

The total normal deformation of the two cylinder surfaces is given by:

\[
u(x) = -\frac{4}{\pi E} \int_s^x p(s) \ln |x - s| \, ds \quad [16]
\]

where \( x_i \) is the inlet coordinate of the film, \( x_o \) is the outlet coordinate, and \( E' \) is the equivalent Young’s modulus.

The grease film thickness equation is given by:

\[
h = h_0 + \frac{x^2}{2R} + \nu(x) - \nu(o) \quad [17]
\]

where \( h_0 \) is the central film thickness.

**BRIEF DESCRIPTION OF NUMERICAL PROCEDURE**

The above equations are nondimensionalized. The dimensionless parameters are listed in the Nomenclature.

The lubricated region is divided into three sub-regions. The forward-iterative method is used in the regions of lower pressure, i.e., inlet and outlet region, and the inverse-iterative method is used in the central region of high pressure. This method is suited to the EHL problem under heavy loads.

The central film thickness and position of the pressure spike are obtained from a continuity condition of pressure on the conjunction points of two regions. A simple method using a floating pressure spike is applied in the process of iteration. In order to limit the pressure fluctuation and obtain convergence for the iterative calculation, under-relaxation factors are introduced.

The revised pressure distribution can be expressed as:

\[
\{ p \}_{k+1} = \{ p \}_k + \{ w \}_k (\Delta p)^k
\]

where \( \{ p \}_k \) is the pressure distribution of last iteration, \( \{ w \}_k \) is the under-relaxation factor matrix, and \( \{ \Delta p \}_k \) is the vector of pressure differences. It can be obtained by the inverse matrix of the numerically discrete matrix for the Eq. [17]

\[
\{ \Delta p \} = [I]^{-1} \{ \Delta h \}
\]

where \([I]^{-1}\) is the inverse matrix of \([I]\). The variable \( \{ \Delta h \} \) is the vector of film thickness difference, i.e. the difference of the thickness obtained by inverse solution of Reynolds equation [15] and the thickness obtained by film thickness equation [17] with the same pressure distribution.

Experimental data for grease flow characteristics were offered by a Chinese oil plant. With a capillary viscometer, experimental data, i.e., shear ratio \( \frac{\partial \mu}{\partial \eta} \) and shear stress \( \tau \), are obtained. From these data, the Herschel-Bulkley models for each grease are formed by a regression fit method. By use of a ln function, the Herschel-Bulkley model is linear, \( \ln(\tau - \tau_o) = \ln(\eta) + n \ln \left( \frac{\partial \mu}{\partial \eta} \right) \), therefore, a linear-regression fit method can be used. The regression fit for one grease curve is shown in Fig. 1 on an \( \ln - \ln \) plot, and the rheological characteristics for each grease, constitutive equation, are shown in Table 1.

**RESULTS AND DISCUSSION**

In order to verify the accuracy of the solution procedure, numerical results on oil film thickness were obtained, which were almost the same as Dowson’s results. Calculated dimensionless velocities \( U \) are in the range from \( 10^{-10} \) to \( 10^{-12} \), dimensionless loads \( W \) from \( 10^{-5} \) to \( 3 \times 10^{-4} \), and dimensionless material parameter \( G \) is 5000. Figure 2 shows the numerical results for oil which justify use of the method to solve EHL for grease.
Since the viscosity of grease and oil are different under the same velocity of the cylinder, film thickness differences must exist. In order to understand the difference between oil and grease film thickness, the grease and oil film thickness difference must be calculated for the same velocity of cylinders. Since the solved equations are dimensionless and the dimensionless velocities of oil, \( U_{oil} \), and grease, \( U_{g} \), are different, the relations between them must be derived. From the condition that the velocity of the cylinder lubricated by oil is the same as that of the cylinder lubricated by grease comes the following equation:

\[
U_{g} = \frac{1}{\eta_{oil}} \frac{\tau_{oil}}{E'} \left(1 - \frac{1}{\eta_{oil}} \right) U_{oil}
\]

Putting \( E' = 2.27 \times 10^{11} \text{ N/m}^2 \) and data from Table 1 in the above equation yields

\[
U_{g} = 2.0 \times 10^{-7} * U_{oil} \quad \text{(Sodium)}
\]
\[
U_{g} = 5.5 \times 10^{-4} * U_{oil} \quad \text{(Lithium)}
\]
\[
U_{g} = 4.0 \times 10^{-4} * U_{oil} \quad \text{(Calcium)}
\]

Thus, under the same velocity of the cylinder, the relation of viscosity of the base oil and viscosity from the Herschel-Bulkley model changes into that of dimensionless velocity. With these corresponding dimensionless velocities, one can compare oil film with grease film under the same velocity of cylinders.

Equation [21], shows that the dimensionless velocity is smaller than that of its base oil under the same velocity of cylinders. This is because the plastic viscosity of grease is larger than the viscosity of its base oil. This will result in film thickness of grease that is larger than that of its base oil.

The following numerical results deal primarily with rolling cylinders, i.e., \( V/U = 0 \). Only Item 3 below deals with approach motion, i.e., \( V/U \neq 0 \).

**Relation Between Velocity and Grease Film Thickness**

In this paper, four kinds of greases are used for calculating film and pressure. The results of minimum and central film thickness under various velocities are shown in Table 2.

Figure 3 shows pressure distribution and film thickness for calcium grease under a series of velocities. It is shown that as velocity increases, film thickness and pressure spike size increase, and the pressure spike moves toward the center of the cylinder.

Figures 4 and 5 show pressure and film for four kinds of grease and one kind of oil under two velocities. It is found that film thickness and pressure spike of grease are greater than those of the base oil. In addition, the grease spike is closer to the center of the cylinder under the same cylinder velocity in full lubrication condition, particular at

### Table 1

<table>
<thead>
<tr>
<th>Grease</th>
<th>Base Oil Viscosity 20°C</th>
<th>Constitutive Equation at 20°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium</td>
<td>0.26</td>
<td>( \tau = 5000 + 8519 \left( \frac{\partial u}{\partial y} \right)^{0.62} )</td>
</tr>
<tr>
<td>Calcium</td>
<td>0.075</td>
<td>( \tau = 2800 + 32.4 \left( \frac{\partial u}{\partial y} \right)^{0.62} )</td>
</tr>
<tr>
<td>Sodium</td>
<td>0.12</td>
<td>( \tau = 1500 + 2694 \left( \frac{\partial u}{\partial y} \right)^{0.47} )</td>
</tr>
<tr>
<td>Ref. (1)</td>
<td>0.13</td>
<td>( \tau = 48 + 1.6 \left( \frac{\partial u}{\partial y} \right)^{0.82} )</td>
</tr>
</tbody>
</table>

### Table 2—Film Thickness \( G = 5000 \ W = 3 \times 10^{-5} \)

<table>
<thead>
<tr>
<th>Lubricant</th>
<th>No. 3 Lithium</th>
<th>No. 3 Sodium</th>
<th>No. 3 Calcium</th>
<th>Presheared Grease</th>
<th>Base Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>U(oil)</td>
<td>( H_{ref}10^{-4} H_{min}10^{-4} H_{gap} ) hrs</td>
<td>( H_{ref}10^{-4} H_{min}10^{-4} H_{gap} ) hrs</td>
<td>( H_{ref}10^{-4} H_{min}10^{-4} H_{gap} ) hrs</td>
<td>( H_{ref}10^{-4} H_{min}10^{-4} H_{gap} ) hrs</td>
<td>( H_{ref}10^{-4} H_{min}10^{-4} H_{gap} ) hrs</td>
</tr>
<tr>
<td>2 \times 10^{-12}</td>
<td>0.286 0.232 3.65</td>
<td>0.234 0.154 2.99</td>
<td>0.219 0.147 2.78</td>
<td>0.0937 0.0745 1.19</td>
<td>0.0783 0.0668 1.0</td>
</tr>
<tr>
<td>4 \times 10^{-12}</td>
<td>0.393 0.270 3.99</td>
<td>0.324 0.220 2.55</td>
<td>0.318 0.228 2.50</td>
<td>0.1372 0.1109 1.0</td>
<td>0.127 0.109 1.0</td>
</tr>
<tr>
<td>6 \times 10^{-12}</td>
<td>0.495 0.370 2.95</td>
<td>0.392 0.270 2.33</td>
<td>0.396 0.29 2.36</td>
<td>0.266 0.195 1.12</td>
<td>0.238 0.207 1.0</td>
</tr>
<tr>
<td>8 \times 10^{-12}</td>
<td>0.581 0.440 2.85</td>
<td>0.445 0.315 2.18</td>
<td>0.459 0.338 2.25</td>
<td>0.388 0.362 1.0</td>
<td>0.388 0.353 1.0</td>
</tr>
<tr>
<td>10^{-11}</td>
<td>0.617 0.446 2.59</td>
<td>0.521 0.423 2.19</td>
<td>0.516 0.421 2.19</td>
<td>0.266 0.195 1.12</td>
<td>0.238 0.207 1.0</td>
</tr>
<tr>
<td>2 \times 10^{-11}</td>
<td>0.874 0.678 2.25</td>
<td>0.670 0.496 1.73</td>
<td>0.708 0.538 1.82</td>
<td>0.388 0.362 1.0</td>
<td>0.388 0.353 1.0</td>
</tr>
<tr>
<td>4 \times 10^{-11}</td>
<td>1.211 0.997 1.94</td>
<td>0.891 0.681 1.43</td>
<td>1.03 0.811 1.65</td>
<td>0.624 0.536 1.0</td>
<td>0.624 0.536 1.0</td>
</tr>
<tr>
<td>6 \times 10^{-11}</td>
<td>1.44 1.240 1.74</td>
<td>1.04 0.982 1.38</td>
<td>1.22 0.988 1.48</td>
<td>0.827 0.713 1.0</td>
<td>0.827 0.713 1.0</td>
</tr>
<tr>
<td>8 \times 10^{-11}</td>
<td>1.81 1.552 1.81</td>
<td>1.18 0.931 1.18</td>
<td>1.39 1.142 1.31</td>
<td>0.995 0.782 0.995</td>
<td>1.000 0.861 1.0</td>
</tr>
</tbody>
</table>

**NOTE:** \( H_{ref}/H_{min} \) is the ratio of central grease film thickness to central base oil film thickness.
low velocity at which the grease is of greater viscosity. This is explained by noting that grease is of greater viscosity than base oil; greater hydrodynamic pressure in the inlet region is produced, therefore, higher grease pressure spike forms in the outlet region to balance the hydrodynamic pressure in the inlet region. It is also found that the greater the plastic viscosity \( \eta_p \) and the exponent of velocity gradient \( n \), the greater the film thickness and pressure spike. The exponent of parameter \( U \) is 0.69 for oil and 0.51 for calcium grease.

Figure 6 shows pressure and film of oil, presheared grease, and a starved lubrication condition. The yield stress and plastic viscosity of presheared grease are small because grease structure is destroyed by mechanical shear. The model for presheared grease in this paper is obtained from Ref. \( (I) \) as \( \tau = 48 + 1.6 \left( \frac{\partial u}{\partial y} \right)^{0.82} \). The film and pressure shapes of presheared grease are similar to those of fresh grease, but the absolute values of film and pressure of fresh grease are greater than those of presheared grease under the same condition. The calculated film thickness ratio of presheared grease to base oil are in the range from 1.2 to 1.0, which are the same as Jonkisz's results. The film thickness ratios of fresh greases to base oil are in the range from 3.6 to 1.2.

The Effects of the Yield Stress on Grease Film

The yield stress has some effect on film thickness at lower velocity when grease yield stress is greater. The calculated values are shown in Table 3. The greater the yield stress, the greater the grease film thickness. The grease film thicknesses are increased by nine percent under low velocity and five percent under medium velocity. Under high velocity, the yield stress has a negligible effect on the film thickness. Greenwood and Jonkisz came to the conclusion that the yield stress has a negligible effect on the film thickness, but the yield stress in their models of grease is small.

The Effects of Velocity Ratio on Grease Film and Pressure

Velocity ratio \( V/U \) has an important effect on the film and pressure when it is in the range from \( 10^{-4} \) to \( 10^{-3} \). The calculated results are shown in Table 4 and Fig. 7.

The approaching motion of two cylinders increases the load capacity for a constant film. In other words, under a constant load, the grease film thickness will be increased. The separating motion of two cylinders reduces the load capacity. The more positive the velocity ratio, the greater the film thick ness and pressure spike, and the closer to the
Table 3—The Effect of Yield Stress on Film

<table>
<thead>
<tr>
<th>$U$</th>
<th>$H_{o}=10^{-4}$</th>
<th>$H_{o}=10^{-4}$</th>
<th>$H_{o}=10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{oo}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.21</td>
<td>0.5043</td>
<td></td>
</tr>
<tr>
<td>7.5 $\times$ 10$^{-9}$</td>
<td>0.216</td>
<td>0.5114</td>
<td></td>
</tr>
<tr>
<td>1.3 $\times$ 10$^{-8}$</td>
<td>0.218</td>
<td>0.5169</td>
<td>0.1394</td>
</tr>
<tr>
<td>2.5 $\times$ 10$^{-8}$</td>
<td>0.231</td>
<td>0.5519</td>
<td>0.1438</td>
</tr>
</tbody>
</table>

Note: $P_{s}$ is the pressure of the pressure spike $X_{s}$ is the point of the pressure spike $H_{o}$ is the thickness when $Ra=0$

Table 4—The Effect of Velocity Ratio on Film and Pressure Spike

$U_{o} = 4 \times 10^{-15}$ $G = 5000$ $W = 3 \times 10^{-5}$

<table>
<thead>
<tr>
<th>$Ra (U/V)$</th>
<th>$H_{o}=10^{-4}$</th>
<th>$H_{min}=10^{-4}$</th>
<th>$P_{s}=10^{-2}$</th>
<th>$X_{s}$</th>
<th>$(H_{o} - H_{o1})/H_{o1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-10^{-3}$</td>
<td>0.3877</td>
<td>0.2190</td>
<td>0.41</td>
<td>0.65</td>
<td>-24.9%</td>
</tr>
<tr>
<td>$-10^{-4}$</td>
<td>0.5080</td>
<td>0.4191</td>
<td>0.381</td>
<td>0.62</td>
<td>-1.63%</td>
</tr>
<tr>
<td>$-10^{-5}$</td>
<td>0.5122</td>
<td>0.4202</td>
<td>0.387</td>
<td>0.61</td>
<td>-0.8%</td>
</tr>
<tr>
<td>0</td>
<td>0.5164</td>
<td>0.4231</td>
<td>0.391</td>
<td>0.61</td>
<td>0</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>0.5212</td>
<td>0.4332</td>
<td>0.390</td>
<td>0.61</td>
<td>0.98%</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>0.5309</td>
<td>0.4451</td>
<td>0.395</td>
<td>0.61</td>
<td>2.81%</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>0.6389</td>
<td>0.5418</td>
<td>0.590</td>
<td>0.54</td>
<td>23.7%</td>
</tr>
</tbody>
</table>

Fig. 7—The effect of ratio on film and pressure spike $G = 5000$ $W = 3 \times 10^{-5}$ $U_{o} = 2 \times 10^{-18}$ $Ra$: 1. $-10^{-3}$ 2. $-10^{-4}$ 3. $10^{-5}$ 4. $10^{-5}$ 5. $10^{-3}$

Mechanism of Starvation and Starved Lubrication Condition

The mechanical elements lubricated by grease are generally fed at regular intervals. Most of the grease is squeezed out of the lubricated region within a short time, therefore, a starved lubrication condition exists in mechanical elements. According to the experiments done by Poon (5) and Aihara (6), a stable film exists between two cylinders, and the film thickness hardly changes even though no grease is fed during a long period of time. With a disk machine made by the author, it is observed that at the beginning, a large amount of grease is fed to the cylinders, but most of it is squeezed out of the contact region within a short time. Some of it adheres to the side of the cylinders to form a side band. Since it is not sheared, and there exists a yield stress, the side band is preserved at the side of the cylinders. It is inferred that the side band can prevent part of the grease from squeezing out of the lubricated region; that there may exist the exchange of grease between the side band and the lubricated region; and that, since a yield stress exists, the flow of grease on the axis of the cylinder is more difficult than that of oil, therefore, part of grease may be easily held in the contact region. As a result, two cylinders are able to hold a stable film for a long time.

In order to calculate the stable film thickness in a starved lubrication condition, the positions of the initial pressure point in the inlet must be known. Many authors (3), (6) have adopted the position that reverse flow vanishes as the beginning point of pressure in a starved lubrication condition. In this paper, two methods are used to determine the beginning point of pressure: one is to adopt the position where reverse flow vanishes as the beginning point of pressure, another is to adopt the experimental results obtained by Poon who found $-1.3a$ as the beginning point of pressure in a disk machine.

Under medium velocity, the calculated results in a starved lubrication condition are shown in Table 5. The calculated ratios ($H_{og}/ H_{oo}$) of grease central films to corresponding base oil central films are between 1.8 and 0.9, which are greater than those of Poon’s experiments, in the vicinity of 0.7, when the reverse flow condition is used as the beginning point. When $-1.3a$ is used for the beginning point of pressure, most of the calculated results of the ratios are between 0.7 and 0.8, confirming Poon’s experiments.
TABLE 5—STARVED GREASE LUBRICATION FOR DIFFERENT CONDITIONS $U_0 = 10^{-11}$  $W = 3 \times 10^{-5}$ $G = 5000$

<table>
<thead>
<tr>
<th>LUBRICANT</th>
<th>NO. 3 SODIUM</th>
<th>NO. 3 CALCIUM</th>
<th>PRESHEARED GREASE</th>
<th>BASE OIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary</td>
<td>$H_1 \times 10^{-4}$ $H_0 \times 10^{-4}$ $H_{min} \times 10^{-4}$</td>
<td>$H_1 \times 10^{-4}$ $H_0 \times 10^{-4}$ $H_{min} \times 10^{-4}$</td>
<td>$H_1 \times 10^{-4}$ $H_0 \times 10^{-4}$ $H_{min} \times 10^{-4}$</td>
<td>$H_1 \times 10^{-4}$ $H_0 \times 10^{-4}$ $H_{min} \times 10^{-4}$</td>
</tr>
<tr>
<td>$X = -1.3a$</td>
<td>1.81 0.174 0.757</td>
<td>0.157 0.150 0.66</td>
<td>0.127 0.116 0.531</td>
<td>0.178 0.165 0.747</td>
</tr>
<tr>
<td>reverse flow boundary</td>
<td>0.421 0.341 1.766</td>
<td>0.446 0.570 1.872</td>
<td>0.207 0.172 0.868</td>
<td></td>
</tr>
<tr>
<td>The point of reverse flow</td>
<td>-2.4a</td>
<td>-1.8a</td>
<td>-1.8a</td>
<td>-1.6a</td>
</tr>
</tbody>
</table>

Under high velocity, it is difficult to measure the beginning point, and no author could give the beginning point by measurement. By adopting $-1.3a$ as the beginning point, the calculated results of the ratios in this paper are between 0.3 and 0.4, refer Fig. 9, which are smaller than those of Aihara's experiments, between 0.5 and 0.7. Since the grease film thickness changes drastically in the vicinity of $-1.3a$, it is important to accurately measure the beginning point in a starved condition to calculate grease film.

Figure 8 shows the calculated results for film thicknesses as they vary with the beginning coordinate of pressure under a dimensionless velocity of $2 \times 10^{-18}$ for sodium soap grease. When the beginning point is farther than $-2.0a$, the difference in beginning points has little effect on film thickness. When the beginning point is closer than $-2.0a$, grease film thicknesses vary rapidly with the beginning coordinates. Under higher velocity ($1.2 \times 10^{-17}$), shown in Fig. 9, when the beginning coordinate is closer than $-3.0a$, it is found that serious starvation of grease occurs and grease film thickness changes rapidly.

Figure 10 shows the pressure and film varying with the beginning coordinate of pressure. When the beginning coordinates move toward the center of the cylinder, the position of the spike moves toward the outlet region, the spike becomes smaller, and the difference between minimum and central film thicknesses becomes smaller. These conclusions are supported by Castle's experiments (7).

The Effect of Different Rheological Models on Calculated Film Thickness

The Bingham and Herschel-Bulkley models are often used to represent grease characteristics. Two curves are generated by Bingham, Fig. 11, and Herschel-Bulkley, Fig. 1, models respectively by means of the regression fit method using the same group of experimental data. Data only fit the Bingham model within the highly sheared region if the data in the mildly sheared region are neglected, curve 1 in Fig. 11. Data fit the Bingham model poorly if considered within the whole sheared region, curve 2 in Fig. 11. Table 6 shows that the calculated grease film thicknesses according to the two models are almost the same at lower velocity, but exhibit some difference at medium velocity.

CONCLUSIONS

1. Fresh grease film thickness is greater than oil film thickness. The ratios of central film thickness for grease to central film thickness for base oil are in the range from 3.6 to 1.2, with larger values at low velocity and
of the yield stress. The calculated grease film in a starved condition is greater than that of the reverse flow condition adopted under medium velocity. When the experimental value $-1.3a$ is adopted as the beginning point of pressure, the calculated ratios of the starved grease central film thickness to the central film thickness of its base oil in a full lubricated condition range from 0.4 to 0.8.

3. The yield stress has some effect on film thickness when the yield stress is great and velocity is low. An approaching motion of the two cylinders increases film thickness and separating motion decreases film thickness.

4. The film thicknesses calculated using the Bingham and Herschel-Bulkley models are almost the same at low velocity and exhibit some difference at medium velocity.

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REFERENCES