Abstract

New computational algorithm has been developed for solving Elastohydrodynamic Lubrication (EHL) problem for infinite line contact which is based on h-p non conforming least square spectral element method (h-p LSSEM) using parallel computers. The Reynolds, Elasticity and force balance equations are solved simultaneously. A systematic approach has been developed and investigated using parallel computers. PCGM solver is used for solving Reynolds equation. Film thickness equation is solved using singular quadrature technique. $H^2$ norm pre-conditioner has been used to enhance the performance of iteration steps. The numerical procedure is stable and accurate enough to capture pressure spike. Comparison of the results have been done to validate the method. Roughness effect also investigated using present method.

Keywords: Elastohydrodynamics Lubrication; h-p non conforming LSSEM; Spectral Element Method

1 Introduction

In the past few years spectral element methods based on least squares variational principles have drawn considerable attention of mathematician and engineers. Spectral Methods are based on global interpolation functions, in contrast to finite difference method (FDM) and finite element method (FEM) which are based on local approximation of functions. The basic principle of this numerical technique is to represent the dependent variable in a finite series of known infinitely differentiable global functions. The series is then substituted into the differential (or integral) equation and upon the minimization of the residual function the unknown coefficients are computed. This approach is similar to DG-FEM used by H. Lu. presented in his paper[10]. We divide our domain in a number of sub-domain and minimize the inter element boundary jumps and their derivatives. This give us automatic stability near the convection dominated region.

2 Governing Equations

The mathematical model of line-contact EHL problems typically consists of three equations shown here in the dimensionless form, described fully in [6]. The Reynolds equation reads

$$\frac{\partial}{\partial X} \left( \varepsilon \frac{\partial P}{\partial X} \right) - \frac{\partial (\rho H)}{\partial X} = 0 \quad (1)$$

where $\varepsilon = \frac{\partial H^3}{\partial X}$, $P$ and $H$ are unknown pressure and film thickness, $\rho$ and $\eta$ are functions of pressure. The boundary conditions for (1) are $P = 0$ at $X = X_{in}$ and $P = \frac{\partial P}{\partial X} = 0$ at $X_{out}$.

Film shape in the dimensionless form at any point $X_i$ is given by

$$H(X_i) = H_0 + \frac{X_i^2}{2} - \frac{1}{\pi} \int_{X_{in}}^{X_{out}} P(X') \ln |X - X'| dX' \quad (2)$$

$H_0$ is constant term.

The dimensionless force balance equation is defined as follows

$$\int_{X_{in}}^{X_{out}} P(X') dX' = \frac{\pi}{2} \quad (3)$$

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2.1 Density and Viscosity

The viscosity-pressure relationship proposed by Roelands is adopted here [8].

\[ \eta_l = \exp\{\ln \eta_o + 9.67[-1 + (1 + 5.1 \times 10^{-9} p_l P)^2]\} \quad (4) \]

where \( Z = \frac{\ln \eta_o + 9.07}{0.5 \times 10^{-9}} \).

The density-pressure relation given by Dowson and Higginson [4] is adopted in this paper in dimensionless form.

\[ \rho_l = \left[1 + \frac{0.6 \times 10^{-9} p_l P}{1 + 1.7 \times 10^{-9} p_l P}\right] \quad (5) \]

3 Solution Procedure

Many methods are available for solving EHL problem Like FD ,FEM , DG-FEM & BEM. In this paper a new method is developed. A brief discussion is given below.

3.1 LSSEM Formulation

Let us divide our domain \( \Omega = [-4.5, 1.5] \) into \( np \) number of sub-domains \( I_1, I_2, ..., I_{np} \). Define a nonconforming spectral element representation on each of these sub domains as follows:

Let \( u_k^{(p_k)} = \sum_{n=0}^{p_k} a_{n}^{k} \phi_{n}^{k} \), \( 1 \leq p_k \leq W \), \( 1 \leq k \leq np \).

We define the analytic map \( M^k \) from the master interval \( S = (-1, 1) \rightarrow I_k \).

Let \( u(M^k(x)) = \sum_{n=0}^{p_k} a_{n}^{k} \phi_{n}^{k} \).

We define the space of spectral element function \( \prod_{0}^{W} = \{u_k^{(p_k)}\} \).

We define residual functional for our problem.

Let \( \{u_k^{(p_k)}, 1 \leq k \leq np\} \in \prod_{0}^{W} \), the space of spectral element functions. Define the functional

\[ \tau^W(\{u\}) = \sum_{k=1}^{np}(\|A(u_k) - F\|_{0, I_k}^2 + \|u\|_{0, \gamma_k}^2) \quad (6) \]

3.2 Residual Computations

The solution of above problem can be obtained by minimizing the functional (6) \( \tau^W(\{v(\Phi^k)\}_k) \) over all \( \{v(\Phi^k)\}_k \).

The solution can be obtained by using the PCGM for solving the normal equations.

Now

\[ \tau^W(U + \epsilon V) = \tau^W(U) + 2\epsilon V^T(SU - TG) + O(\epsilon^2) \forall V \quad (7) \]

Where \( U & V \) are a vector assembled from the value of \( \{u_k^{(p_k)}(\phi_{m}), 1 \leq k \leq np, 1 \leq m \leq p_j\} \).

Thus as by minimization property we need to solve the given system \( SU - TG = 0 \) The residuals in the normal equations can be calculated. We collocate the partial differential equation on a finer grid of Gauss-Lobatto-Legendre points.

Then we apply the adjoint differential operator to these residuals and project these values back to the original grid.

3.3 Convergence criterion

Following convergence criterion is used for present study:

\[ E_p = \frac{\|P^{n+1} - P^n\|_0}{\|P^n\|_0} < \text{tolerance} \]

3.4 Film-thickness Calculation

\[ h(x) = h_0 + \frac{x^2}{2} - \frac{1}{\pi} \int_{X_{in}}^{X_{out}} \ln |x - x'| p(x') dx' \]

Using the spectral representation of \( p(x') \), we get

\[ h(x) = h_0 + \frac{x^2}{2} - \frac{1}{\pi} \sum_{c=1}^{N} \int_{c}^{N} \ln |x - x'| \sum_{i=0}^{n+1} a_i g_i'(x') dx' \]
This equation can be simplified to read
\[ h(x) = h_0 + \frac{x^2}{2} - \frac{1}{\pi} \sum_{e=1}^{N} \sum_{i=0}^{p_e+1} K^e_i(x) a^e_i \]
(8)

Where the kernel values \( K^e_i \) are defined by:
\[ K^e_i = \int_{x} \ln |x - x'| g^e_i(x') dx' \]
(9)

when \( x \) is inside of \( e \) (the element value) it has a weak singularity which is handled using singular quadrature, otherwise Gaussian quadrature is employed.

3.5 Load Balance Equation Calculation

The force balance equation is discritized according to:
\[ \sum_{e=1}^{N} \int_{x}^{x_0} g^e_i(x) a^e_i dx - \frac{\pi}{2} = 0 \]
(10)

By introducing another kernel \( gg^e_i \)
\[ gg^e_i = \int_{x} g^e_i(x) dx \]
(11)

the discrete force balance equation can be rewritten as:
\[ \sum_{e=1}^{N} \sum_{i=0}^{p_e+1} (gg^e_i) a^e_i - \frac{\pi}{2} = 0 \]
(12)

4 Numerical Results

Surface roughness (deterministic) can be incorporated by modifying the film thickness
\[ h(x) = h_0 + \frac{x^2}{2} - \frac{1}{\pi} \int_{X_{in}}^{X_{out}} p(x') \ln |x - x'| dx' + \text{Asin} \left( \frac{2\pi X}{\lambda^*} \right) \]
(13)

We have taken following parametric values for our case study: \( U = 7.3 \times 10^{-11}, \alpha = 1.59 \times 10^{-8}, \lambda^* = 0.125, R_e = 2.0 \times 10^{-2}, G = 3500, \eta_0 = 0.04, W = 10^{-4}, X_{in} = -4.0, X_{out} = 1.5 \)

5 Conclusion

It is always not easy to get solution of EHL problem for all range of parameter so homotopy approach is applied. The concept of a homotopy is simple. It is the deformation of one problem into another by the continuous variation of a single parameter. The key idea here is that one of the the problems will be easy to solve and this will be continuously deformed into one that is hard to solve. The results obtained are in conformity with the existing results. We are also trying to extend our idea for point contact case (two-Dimensional case). Surface roughness case will also study in point contact case also.

References
Figure 1: Pressure and Filmthickness profile plot


KEYWORDS: Computation: Fluid Mechanics Methods, Boundary Lubrication: Boundary Lubrication (General), EHL: Partial-EHL, Roughness Effects.